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Additional Conservation Laws for Two-Velocity Hydrodynamics Equations with the Same Pressure in Components

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Abstract: *A series of the differential identities connecting velocities, pressure and body force in the two-velocity hydrodynamics equations with equilibrium of pressure phases in reversible hydrodynamic approximation is obtaned.*
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I. Introduction

Mathematical model of multiphase and multicomponent media are being built by using conservation laws. The conservation laws approach assumes that all phases and components, including particulate media modeled as a continuous medium, each of which is formally distributed throughout the computational domain. Thus at each point in the consedered domain all the parameters of each phase (continuum) are formally defined. The big advantage of a mathematical model based on the method of conservation laws is the physical correctness of the received systems of differential equations. Multiphase and multicomponent media with phase pressure equilibrium occur in the petroleum and chemical industry, energetics and other fields.

The study of flow of viscous compressible / incompressible liquids based on the solutions to two-speed complete hydrodynamic equations is of great relevance. As known from the literature, there are very limited number of cases admitting analytic integration of the Navier-Stokes equations [1]. In [2] a description of the flow of an incompressible two-speed viscous liquid for the case of phase pressure equilibrium at constant volume saturation substances is giveng by using scalar functions. A system of differential equations for these functions is obtaned also fundamental solution to the system in the case of the three-dimensional stationary flows of viscous two-speed continuum with the phase pressure equilibrium is built in [3]. These solutions can be useful for testing of numerical methods for solving the two-velocity hydrodynamics equations.

In vector analysis, the field theory and mathematical physics, the classical differential identities are played very important role. In [4] a series of formulas of vector analysis in the form of the differential identities of the second and third order connecting a Laplacian of arbitrary smooth scalar function of two independent variables $u(x, y)$, the module of a gradient of this function, angular value and the direction of a gradient is obtained. The results of [4] are generalized in [5] in two ways: for a three-dimensional case and for arbitrary (not necessarily potential) smooth vector field $\mathbf{v}$. A series of formulas of vector analysis in the form of differential identities which, on the one hand, connecting the module $|\mathbf{v}|$ and the direction $\boldsymbol{\tau}$ of an arbitrary smooth vector field $\mathbf{v} = |\mathbf{v}|\boldsymbol{\tau}$ in three-dimensional $(\mathbf{v} = \mathbf{v}(x, y, z))$ and in two-dimensional $(\mathbf{v} = \mathbf{v}(x, y))$ cases is taken. On the other hand, these formulas separate the module $|\mathbf{v}|$ and the direction $\boldsymbol{\tau}$ of a vector field $\mathbf{v} = |\mathbf{v}|\boldsymbol{\tau}$. Namely, the main identity compares any smooth vector field $\mathbf{Q} = \mathbf{P} + \mathbf{S}$, where the field $\mathbf{P}$ is defined only by the module $|\mathbf{v}|$ of the field $\mathbf{v}$ and is potential both in two-dimensional and in three-dimensional cases, and the field $\mathbf{S}$ is defined only by the direction $\boldsymbol{\tau}$ of the field and is solenoidal in a two-dimensional case. Applications of the obtaned identities to the Euler hydrodynamic equations are given.

In this work, additional conservation laws for the equations of two-speed hydrodynamics with one pressure are obtaned.

II. Auxiliary Statements

In [5], A. G. Megrabov has received important differential identities connecting the module and the direction of a vector field. Let us provide them.

Theorem 1. For any vector feild $(\mathbf{v} = \mathbf{v}(x, y, z)) = |\mathbf{v}|\boldsymbol{\tau}$ with the componrnts $\vartheta_k(x, y, z) \in C^1(D)$, $k = 1, 2, 3$, modul $|\mathbf{v}| \neq 0$ in D and direction τ the following identety holds

$$\mathbf{Q} = \mathbf{Q}(\mathbf{v}) = \mathbf{P}(|\mathbf{v}|) + \mathbf{S}(\boldsymbol{\tau}), \tag{1}$$

where

$$\mathbf{Q}(\mathbf{v}) \overset{\text{def}}{=} \frac{\mathbf{v}\,\mathrm{div}\mathbf{v} + \mathbf{v}\times\mathrm{rot}\mathbf{v}}{|\mathbf{v}|^2}, \quad \mathbf{P}(\mathbf{v}) \overset{\text{def}}{=} \nabla \ln|\mathbf{v}| = \frac{\nabla|\mathbf{v}|}{|\mathbf{v}|}, \tag{2}$$

$$\mathbf{S} = \mathbf{S}(\boldsymbol{\tau}) \overset{\text{def}}{=} \boldsymbol{\tau}\mathrm{div}\boldsymbol{\tau} + \boldsymbol{\tau}\times\mathrm{rot}\boldsymbol{\tau} = \mathbf{Q}(\mathbf{v}) - \mathbf{P}(|\mathbf{v}|). \tag{3}$$

For the vector field $\mathbf{S}$ any of the following representations takes place

$$\mathbf{S} = \mathbf{S}(\boldsymbol{\tau}) = \boldsymbol{\tau}\mathrm{div}\boldsymbol{\tau} - \boldsymbol{\tau}_s = -\{(\boldsymbol{\tau}\times\nabla)\times\boldsymbol{\tau} + (\boldsymbol{\tau}\cdot\nabla)\boldsymbol{\tau}\} = -\frac{(\mathbf{v}\times\nabla)\times\mathbf{v}}{|\mathbf{v}|^2} \tag{4}$$

($\boldsymbol{\tau}_s = (\boldsymbol{\tau}\cdot\nabla)\boldsymbol{\tau} = \mathrm{rot}\boldsymbol{\tau}\times\boldsymbol{\tau}$ is the derivative of the vector $\boldsymbol{\tau}$ in the direction $\boldsymbol{\tau}$),

$$\mathbf{S} = \mathrm{rot}(\alpha\mathbf{k}) - \cos^2\theta\,\mathrm{rot}(\alpha\mathbf{k} - \mathrm{tg}\,\theta\boldsymbol{\lambda}) = \mathrm{rot}(\alpha\mathbf{k} + \cos\theta\boldsymbol{\psi}) - 2\cos\theta\,\mathrm{rot}\boldsymbol{\psi}, \tag{5}$$

where $\boldsymbol{\lambda} = -\sin\alpha\,\mathbf{i} + \cos\alpha\,\mathbf{j}$, $\boldsymbol{\psi} = -\sin\theta\boldsymbol{\lambda} + \alpha\cos\theta\mathbf{k}$,

$$\mathbf{S} = -\nabla\alpha\times(\cos\theta\boldsymbol{\tau} - \mathbf{k}) + \nabla\theta\times\boldsymbol{\lambda}, \quad \mathbf{S} = \boldsymbol{\tau}\mathrm{div}\boldsymbol{\tau} - \kappa\boldsymbol{\nu}, \tag{6}$$

where κ is the curvature of the field line of the field $\mathbf{v}$, $\boldsymbol{\nu}$ is its main normal. For κ we have the formula $\kappa^2 = \sin^2\theta\alpha_s^2 + \theta\alpha_s^2$, where $\alpha_s = (\nabla\alpha\cdot\boldsymbol{\tau})$, $\theta_s = (\nabla\theta\cdot\boldsymbol{\tau})$ are the derivatives of the angles α, θ in the direction $\boldsymbol{\tau}$.

The main identety (1) can also be rewritten in the following form

$$\mathbf{Q} + \mathbf{H}_i = \nabla\ln|\mathbf{v}| + \mathrm{rot}\mathbf{F}_i, \quad i = 1, 2,$$

where $\mathbf{H}_1 = \cos^2\theta\,\mathrm{rot}(\alpha\mathbf{k} - \mathrm{tg}\,\theta\boldsymbol{\lambda})$, $\mathbf{H}_2 = 2\cos\theta\,\mathrm{rot}\psi$, $\mathbf{F}_1 = \alpha\mathbf{k}$, $\mathbf{F}_2 = \alpha\mathbf{k} + \cos\theta\psi$, so that vectors $\mathbf{H}_i, \mathbf{F}_i$, and $\mathbf{S}$ are determined only by angles α, θ, that is by the direction $\boldsymbol{\tau}$ of the field $\mathbf{v}$.

If the property $|\mathbf{v}| \neq 0$ in D is not assumed, then identety (1) takes the form

$$\mathbf{W} = \mathbf{v}\mathrm{div}\mathbf{v} + \mathbf{v}\times\mathrm{rot}\mathbf{v} = \nabla|\mathbf{v}|^2 - \mathbf{V},$$

where

$$\mathbf{V} \overset{\text{def}}{=} -|\mathbf{v}|^2\mathbf{S} = \frac{1}{2}\nabla|\mathbf{v}|^2 - \mathbf{v}\mathrm{div}\mathbf{v} - \mathbf{v}\times\mathrm{rot}\mathbf{v} = -|\mathbf{v}|^2\{\boldsymbol{\tau}\mathrm{div}\boldsymbol{\tau} + \boldsymbol{\tau}\times\mathrm{rot}\boldsymbol{\tau}\} = \mathbf{v}\times\nabla\times\mathbf{v}.$$

Other formulas for $\mathbf{W}, \mathbf{V}$ can be derived by substituting any expression for $\mathbf{S}$ from (4)-(6) in the last equalities.

Theorem 2. On conditions of theorem 1 and $\upsilon_k(x, y, z) \in C^2(D)$ $(k = 1, 2, 3)$, we have

$$\mathrm{div}\mathbf{S} = -2\sin\theta(\boldsymbol{\tau}\cdot\mathbf{B}) = -\frac{2\sin\theta(\mathbf{v}\cdot\mathbf{B})}{|\mathbf{v}|},$$

where $\mathbf{B} = \nabla\alpha\times\nabla\theta = \mathrm{rot}(\alpha\nabla\theta) = -\mathrm{rot}(\theta\nabla\alpha)$. In addition, the identity

$$\mathrm{div}(\mathbf{Q} - \mathbf{P} + \mathbf{H}_i) = 0 \Leftrightarrow \mathrm{div}\left\{\frac{\mathbf{v}\mathrm{div}\mathbf{v} + \mathbf{v}\times\mathrm{rot}\mathbf{v}}{|\mathbf{v}|^2} - \nabla\ln|\mathbf{v}| + \mathbf{H}_i\right\} = 0, \quad (i = 1, 2)$$

takes place which can be considered as a conservation law (its differential form) with an integral form for the stream

$\iint_S ([\mathbf{Q} - \mathbf{P} + \mathbf{H}_i]\cdot\boldsymbol{\eta})\,dS = 0$, where S is piecewise smooth boundary of the domain D with a normal $\boldsymbol{\eta}$.

In theorems 1 and 2 the followings denotations are accepted: characters $(\mathbf{a}\cdot\mathbf{b})$ and $(\mathbf{a}\times\mathbf{b})$ designate scalar and vectorial product of vectors $\mathbf{a}$ and $\mathbf{b}$; ∇ is Hamiltonian operator (nabla); Δ is Laplace operator; D is some domain in the space of x, y, z; $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along the x-, y-, and z- axes of a rectangular Cartesian coordinate system respectivele; $\mathbf{v} = \mathbf{v}(x, y, z) = \upsilon_1 \mathbf{i} + \upsilon_2 \mathbf{j} + \upsilon_3 \mathbf{k}$ is a vector field on the domain D ; $\upsilon_k = \upsilon_k(x, y, z)$ are scalar functitions, $k = 1, 2, 3$, $|\mathbf{v}|^2 = \upsilon_1^2 + \upsilon_2^2 + \upsilon_3^2$; $\alpha = \alpha(x, y, z)$ is angle of slope of the vector $(\upsilon_1 \mathbf{i} + \upsilon_2 \mathbf{j})$ to the x-axes so that $\cos\alpha = \frac{\upsilon_1}{\sqrt{g}}, \sin\alpha = \frac{\upsilon_2}{\sqrt{g}}$, where $g = \upsilon_1^2 + \upsilon_2^2$ and $\alpha(x, y, z)$ is the polar angel of the point $(\xi = \upsilon_1, \varsigma = \upsilon_2)$ on the plane ξ, ς; $\theta = \theta(x, y, z)$ is the angel between the vector $\mathbf{v}$ and z - axes: $\theta \overset{def}{=} \arccos \frac{\upsilon_3}{|\mathbf{v}|}$ so that $0 \le \theta \le \pi$, $\cos\theta = \frac{\upsilon_3}{|\mathbf{v}|}$, $\sin\theta = \frac{\sqrt{g}}{|\mathbf{v}|}$ (that is α, θ – sferical coordinates in the space $\xi = \upsilon_1, \varsigma = \upsilon_2, \zeta = \upsilon_3$. At the same time $\mathbf{v} = |\mathbf{v}|\boldsymbol{\tau}$, where $\boldsymbol{\tau} = \boldsymbol{\tau}(\alpha, \theta) = \cos\alpha \sin\theta \mathbf{i} + \sin\alpha \sin\theta \mathbf{j} + \cos\theta \mathbf{k}$ is the diraction vector of the vector field $\mathbf{v}$ $(|\boldsymbol{\tau}| = 1)$.

In two dimentional case we have

$\mathbf{v} = \mathbf{v}(x, y) = \upsilon_1 \mathbf{i} + \upsilon_2 \mathbf{j} = \mathbf{v}|\boldsymbol{\tau}|, \upsilon_3 \equiv 0,\ \theta \equiv \frac{\pi}{2} \Rightarrow \boldsymbol{\tau} = \boldsymbol{\tau}(\alpha) = \cos\alpha \mathbf{i} + \sin\alpha \mathbf{j}$, where the angel α is defined by (7), $\nabla\theta = \mathbf{B} = 0$; for any $\varphi(x, y) \in C^1(D)$ we have $\mathrm{rot}(\varphi \mathbf{k}) = \varphi_y \mathbf{i} - \varphi_x \mathbf{j}$, where $\varphi_x = \frac{\partial\varphi}{\partial x}$.

It follows from theorem 1 that

Theorem 3. For any two-dimensional field $\mathbf{v}(x, y)$ with the components $\upsilon_k(x, y) \in C^1(D), k = 1, 2$, with the module $|\mathbf{v}| \ne 0$ in the domain D and with the direction $\boldsymbol{\tau} = \boldsymbol{\tau}(\alpha)$ the following identety holds

$$\mathbf{Q} \overset{def}{=} \frac{\mathbf{v}\,\mathrm{div}\mathbf{v} + \mathbf{v} \times \mathrm{rot}\mathbf{v}}{|\mathbf{v}|^2} = \nabla \ln|\mathbf{v}| + \mathrm{rot}(\alpha \mathbf{k}) \Rightarrow \qquad (8)$$

$$\mathrm{div}\mathbf{v} = \left(\{\nabla \ln|\mathbf{v}| + \mathrm{rot}(\alpha\mathbf{k})\} \cdot \mathbf{v}\right),\quad \mathrm{rot}\mathbf{v} = \{\nabla \ln|\mathbf{v}| + \mathrm{rot}(\alpha\mathbf{k})\} \times \mathbf{v} + \frac{\mathbf{v}(\mathbf{v}\mathrm{rot}\mathbf{v})}{|\mathbf{v}|^2},$$

In additation, $\mathbf{S} = \mathrm{rot}(\alpha\mathbf{k}) \Rightarrow (\mathbf{S} \cdot \nabla\alpha) = 0$, that is field vector lines of the vector field $\mathbf{S}$ coincide with the level lines of the scalar field of polar angels $\alpha(x, y)$. Moreover, if $\upsilon_k(x, y) \in C^2(D)$, $k = 1, 2$, then

$$\mathrm{div}\mathbf{S} = 0, \qquad \mathrm{rot}\mathbf{S} = -(\Delta\alpha)\mathbf{k} \Rightarrow$$

$$\Delta \ln|\mathbf{v}| = \mathrm{div}\mathbf{Q},\quad (\Delta\alpha)\mathbf{k} = -\mathrm{rot}\mathbf{Q} \Rightarrow$$

$$\Delta \mathrm{Ln}\left\{|\mathbf{v}|e^{\pm i\alpha}\right\} = \mathrm{div}\mathbf{Q} \mp i(\mathrm{rot}\mathbf{Q} \cdot \mathbf{k}) \quad (i^2 = -1).$$

In the conservation law of the theorem 2 we have $\mathbf{H}_i = 0, i = 1, 2$,

As it is well-known [6], any smooth vector field can be expressed as the sum of the gradient of some scalar and the rotor of some vector. The identity (8) gives such the expression for the vector field $\mathbf{Q}$. When $\mathbf{v} = \nabla u(x, y)$, theorem 3 impleis the identities obtained in [4].

III. The Equations Of Two-Speed Hydrodynamics With Pressure Equilibrium In Components And Additional Conservation Laws

In [7], on the basis of conservation laws, invariance of the equations with respect to Galilei transformations and conditions of thermodynamic coherence the non-linear two-speed model of fluid flow through a deformable porous medium is constructed**.** Equations of motion of two-speed medium with one pressure in the isothermic case have the form [7,8]

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0,\quad \frac{\partial\tilde\rho}{\partial t}+\operatorname{div}(\tilde\rho\tilde{\mathbf{v}})=0,$$
$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v},\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}+\frac{\tilde\rho}{2\bar\rho}\nabla(\tilde{\mathbf{v}}-\mathbf{v})^2+\mathbf{f},\qquad (9)$$
$$\frac{\partial\tilde{\mathbf{v}}}{\partial t}+(\tilde{\mathbf{v}},\nabla)\tilde{\mathbf{v}}=-\frac{\nabla p}{\tilde\rho}-\frac{\rho}{2\bar\rho}\nabla(\tilde{\mathbf{v}}-\mathbf{v})^2+\mathbf{f},$$

where $\tilde{\mathbf{v}}$ and $\mathbf{v}$ are the speed vectors of components forming a two-speed continuum with partial densiteis $\tilde\rho$ and ρ .; $\bar\rho=\tilde\rho+\rho$ is total density of the continuum; $\mathbf{f}$ is mass force vector carried to a mass unit. The equation of state of the continuum closes system of differential equations (9) and is given by the equation of state

$$p=p\left(\bar\rho,(\tilde{\mathbf{v}}-\mathbf{v})^2\right).$$

It is convenient to enter new pressure

$$\tilde p=p\left(\bar\rho,(\tilde{\mathbf{v}}-\mathbf{v})^2\right)-\frac{\tilde\rho}{2}(\tilde{\mathbf{v}}-\mathbf{v})^2.$$

In the terms of $\tilde p$, p the last two equvation of the system (9) can be transformed in the form

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v},\nabla)\mathbf{v}=-\frac{1}{\bar\rho}\nabla\tilde p-\frac{(\tilde{\mathbf{v}}-\mathbf{v})^2}{2\bar\rho}\nabla\tilde\rho+\mathbf{f},\qquad (10)$$
$$\frac{\partial\tilde{\mathbf{v}}}{\partial t}+(\tilde{\mathbf{v}},\nabla)\tilde{\mathbf{v}}=-\frac{1}{\tilde\rho}\nabla p+\frac{\rho}{\bar\rho\tilde\rho}\nabla\tilde p+\frac{\rho(\tilde{\mathbf{v}}-\mathbf{v})}{2\tilde\rho}\nabla\ln\tilde\rho+\mathbf{f},\qquad (11)$$

In the terms of vectors $\mathbf{W},\mathbf{V},\mathbf{S},\mathbf{Q},\mathbf{P},\mathbf{H}_i,\mathbf{F}_i,\tilde{\mathbf{W}},\tilde{\mathbf{V}},\tilde{\mathbf{S}},\tilde{\mathbf{Q}},\tilde{\mathbf{P}},\tilde{\mathbf{H}}_i,\mathbf{F}_i$ defind in the theorem 1, the system of equvations (10), (11) can be written down in any of the following forms (symbols without tilde and with a tilde fall into to the corresponding components of the continuum):

$$\mathbf{W}=\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\operatorname{div}\mathbf{v}+\frac{1}{2}\nabla\upsilon^2+\frac{1}{\bar\rho}\nabla\tilde p+\frac{(\tilde{\mathbf{v}}-\mathbf{v})^2}{2\bar\rho}\nabla\tilde\rho-\mathbf{f},$$
$$-\mathbf{V}=\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\operatorname{div}\mathbf{v}+\frac{1}{\bar\rho}\nabla\tilde p+\frac{(\tilde{\mathbf{v}}-\mathbf{v})}{2\tilde\rho}\nabla\tilde\rho-\mathbf{f},\qquad (12)$$
$$\mathbf{G}\overset{\text{def}}{=}\frac{1}{\upsilon^2}\left\{\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\operatorname{div}\mathbf{v}+\frac{1}{\bar\rho}\nabla\tilde p+\frac{(\tilde{\mathbf{v}}-\mathbf{v})}{2\tilde\rho}\nabla\tilde\rho-\mathbf{f}\right\}=\mathbf{S}\,(=\mathbf{Q}-\mathbf{P})\Leftrightarrow\qquad (13)$$
$$\Leftrightarrow\ \mathbf{G}+\mathbf{H}_i=\operatorname{rot}\mathbf{F}_i,\quad i=1,2\ .$$
$$\tilde{\mathbf{W}}=\frac{\partial\tilde{\mathbf{v}}}{\partial t}+\tilde{\mathbf{v}}\operatorname{div}\tilde{\mathbf{v}}+\frac{1}{2}\nabla\vartheta^2+\frac{1}{\tilde\rho}\nabla p-\frac{\rho}{\bar\rho\tilde\rho}\nabla\tilde p-\frac{\rho(\tilde{\mathbf{v}}-\mathbf{v})^2}{2\bar\rho}\nabla\ln\tilde\rho-\mathbf{f},$$
$$-\tilde{\mathbf{V}}=\frac{\partial\tilde{\mathbf{v}}}{\partial t}+\tilde{\mathbf{v}}\operatorname{div}\tilde{\mathbf{v}}+\frac{1}{\tilde\rho}\nabla p-\frac{\rho}{\bar\rho\tilde\rho}\nabla\tilde\rho-\frac{\rho(\tilde{\mathbf{v}}-\mathbf{v})}{2\bar\rho}\nabla\ln\tilde\rho-\mathbf{f},\qquad (14)$$
$$\tilde{\mathbf{G}}\overset{\text{def}}{=}\frac{1}{\tilde\upsilon^2}\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial t}+\tilde{\mathbf{v}}\operatorname{div}\tilde{\mathbf{v}}+\frac{1}{\tilde\rho}\nabla p-\frac{\rho}{\bar\rho\tilde\rho}\nabla\tilde p-\frac{\rho(\tilde{\mathbf{v}}-\mathbf{v})^2}{2\bar\rho}\nabla\ln\tilde\rho-\mathbf{f}\right\}=\tilde{\mathbf{S}}\left(=\tilde{\mathbf{Q}}-\tilde{\mathbf{P}}\right)\Leftrightarrow\qquad (15)$$
$$\Leftrightarrow\ \tilde{\mathbf{G}}+\tilde{\mathbf{H}}_i=\operatorname{rot}\tilde{\mathbf{F}}_i,\quad i=1,2\ .$$

In the case of absence of mass forces $\mathbf{f}=0$, the system (9) has the solytion $\mathbf{v}=0$, $\tilde{\mathbf{v}}=0$, $\rho=\rho^0$, $\tilde\rho=\tilde\rho^0$, $p=p^0$ for the liquids in a state of rest with the common pressure $p=p^0$. When the components are homogeneous and incompressible, we have $\rho = const, \tilde{\rho} = const$. Therefore,

$$\mathrm{div}\mathbf{v} = 0, \quad \mathrm{div}\tilde{\mathbf{v}} = 0 \Leftrightarrow \mathbf{v} = \mathrm{rot}\mathbf{A}, \ \ \mathrm{v} = \mathrm{rot}\tilde{\mathrm{A}},$$

where $\mathbf{A}, \tilde{\mathbf{A}}$ are corresponding vector potentials of the speeds $\mathbf{v}, \tilde{\mathbf{v}}$. In other words the vectors $\mathbf{v}, \tilde{\mathbf{v}}$ are solenoidal. In this case the equvations of two-velocity hydrodynamicss can be represented in the form

$$\mathbf{W} = \nabla\left\{\frac{1}{2}\upsilon^2 + \frac{1}{\overline{\rho}}\tilde{\mathrm{p}} + U\right\} + \mathrm{rot}\{\mathbf{A}_t + \mathbf{M}\},$$

$$-\mathbf{V} = \nabla\left\{\frac{1}{\overline{\rho}}\tilde{\mathrm{p}} + U\right\} + \mathrm{rot}\{\mathbf{A}_t + \mathbf{M}\},$$

$$\tilde{\mathbf{W}} = \nabla\left\{\frac{1}{2}\upsilon^2 + \frac{1}{\rho}\tilde{\mathrm{p}} - \frac{1}{\overline{\rho}\tilde{\rho}}\tilde{\mathrm{p}} + U\right\} + \mathrm{rot}\left\{\tilde{\mathbf{A}}_t + \mathbf{M}\right\}$$

$$-\tilde{\mathbf{V}} = \nabla\left\{\frac{1}{\tilde{\rho}}\mathrm{p} + \frac{1}{\overline{\rho}\tilde{\rho}}\tilde{p} + U\right\} + \mathrm{rot}\{\mathbf{A}_t + \mathbf{M}\},$$

where $-\mathbf{f} = \nabla U + \mathrm{rot}\mathbf{M}$; $\tilde{\mathbf{A}}_t, \mathbf{A}_t$ are the derivatives of vectors $\tilde{\mathbf{A}}, \mathbf{A}$ with respect to time. It follows that when the velocities and physical densities of components are the same we have $\tilde{\mathbf{W}} = \mathbf{W}, \tilde{\mathbf{V}} = \mathbf{V}$ and as a result the formulas for the vector fields $\mathbf{W}, \mathbf{V}$.

Thus, for the solution $(\mathbf{v}, \tilde{\mathbf{v}}, \mathrm{p})$ to the two-speed hydrodynamics equations for the homogeneous incompressible liquids can be applied theorem 2.

From (13), (15) and theorem 2 we get

Theorem 4. For any flow of two-speed medium consisting of two incompressible components with the same pressure $(\mathbf{v} \neq 0, \tilde{\mathbf{v}} \neq 0)$ the followng identities take place

$$\mathrm{div}\left[\frac{1}{\upsilon^2}\left\{\frac{\partial \mathrm{v}}{\partial \mathrm{t}} + \mathbf{v}\mathrm{div}\mathbf{v} + \frac{1}{\overline{\rho}}\nabla\tilde{\mathrm{p}} + \frac{(\tilde{\mathbf{v}} - \mathbf{v})^2}{2\overline{\rho}}\nabla\tilde{\rho} - \mathbf{f}\right\}\right] = -2\frac{\sin\theta}{\upsilon}\left(\mathbf{v}\cdot(\nabla\alpha \times \nabla\theta)\right) = \mathrm{div}\mathbf{S},$$

$$\mathrm{div}\left[\frac{1}{\upsilon^2}\left\{\frac{\partial \tilde{\mathbf{v}}}{\partial \mathrm{t}} + \tilde{\mathbf{v}}\mathrm{div}\tilde{\mathbf{v}} + \frac{1}{\tilde{\rho}}\nabla\mathrm{p} - \frac{\rho}{\overline{\rho}\tilde{\rho}}\nabla\tilde{\mathrm{p}} - \frac{\rho(\tilde{\mathbf{v}} - \mathbf{v})^2}{2\tilde{\rho}}\nabla\ln\tilde{\rho} - \mathbf{f}\right\}\right] =$$

$$= -2\frac{\sin\theta}{\tilde{\vartheta}}\left(\tilde{\mathbf{v}}\cdot\left(\nabla\tilde{\alpha} \times \nabla\tilde{\theta}\right)\right) = \mathrm{div}\tilde{\mathbf{S}}.$$

Moreover, besides the common conservation laws for smooth vector fields stated in theorem 2, the conservation laws of differential forms

$$\mathrm{div}(\mathbf{G} + \mathbf{H}_i) = 0, \ \mathrm{div}\left(\tilde{\mathbf{G}} + \tilde{\mathbf{H}}_i\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \mathrm{div}\left[\frac{1}{\upsilon^2}\left\{\frac{\partial \mathbf{v}}{\partial \mathrm{t}} + \mathbf{v}\mathrm{div}\mathbf{v} + \frac{1}{\overline{\rho}}\nabla\tilde{\mathrm{p}} + \frac{(\tilde{\mathbf{v}} - \mathbf{v})^2}{2\overline{\rho}}\nabla\tilde{\rho} - \mathbf{f}\right\} + \mathbf{H}_i\right] = 0,$$

$$\mathrm{div}\left[\frac{1}{\tilde{\upsilon}^2}\left\{\frac{\partial \tilde{\mathbf{v}}}{\partial \mathrm{t}} + \tilde{\mathbf{v}}\mathrm{div}\tilde{\mathbf{v}} + \frac{1}{\tilde{\rho}}\nabla\mathrm{p} - \frac{\rho}{\overline{\rho}\tilde{\rho}}\nabla\tilde{\mathrm{p}} - \frac{\rho(\tilde{\mathbf{v}} - \mathbf{v})^2}{2\overline{\rho}}\nabla\ln\tilde{\rho} - \mathbf{f}\right\} + \tilde{\mathbf{H}}_i\right] = 0$$

and integral forms

$$\iint_S \left([\mathbf{G} + \mathbf{H}_i]\cdot\boldsymbol{\eta}\right)\mathrm{d}S = 0, \ \iint_S \left(\left[\tilde{\mathbf{G}} + \tilde{\mathbf{H}}_i\right]\cdot\boldsymbol{\eta}\right)\mathrm{d}S = 0, \ i = 1, 2.$$

are valid; here the vectors $\mathbf{H}_i(\tilde{\mathbf{H}}_i)$ defined in theorem 1 depend only on the angles of directions of velosities $\mathbf{v}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$, $\tilde{\mathbf{v}}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$; S is piecewise smooth boindary of the domain D; $\boldsymbol{\eta}$ is a unit normal to the S.

In the irrotational motion (as $\mathbf{v} = \nabla u, \tilde{\mathbf{v}} = \nabla \tilde{u}$) case denoting

$$\mathbf{G} \overset{def}{=} \frac{1}{\upsilon^2}\left\{\nabla u_t + \Delta u \nabla u + \frac{1}{\overline{\rho}}\nabla \tilde{p} + \frac{(\nabla \tilde{u} - \nabla u)^2}{2\overline{\rho}}\nabla \tilde{\rho} - \mathbf{f}\right\},$$

$$\tilde{\mathbf{G}} \overset{def}{=} \frac{1}{\tilde{\upsilon}^2}\left\{\nabla \tilde{u}_t + \Delta \tilde{u} \nabla \tilde{u} + \frac{1}{\tilde{\rho}}\nabla p - \frac{\rho}{\overline{\rho}\tilde{\rho}}\nabla \tilde{p} + \frac{\rho(\nabla \tilde{u} - \nabla u)^2}{2\overline{\rho}}\nabla \ln \tilde{\rho} - \mathbf{f}\right\},$$

we have

$$\operatorname{div}\mathbf{G} = \frac{2}{\upsilon}\operatorname{div}\left\{u \operatorname{rot}(\alpha \nabla \cos\theta)\right\} = -\frac{2\sin\theta}{\upsilon}\frac{\partial(u,\alpha,\theta)}{\partial(x,y,z)},$$

$$\operatorname{div}\tilde{\mathbf{G}} = \frac{2}{\tilde{\upsilon}}\operatorname{div}\left\{\tilde{u} \operatorname{rot}(\tilde{\alpha} \nabla \cos\tilde{\theta})\right\} = -\frac{2\sin\tilde{\theta}}{\tilde{\upsilon}}\frac{\partial(\tilde{u},\tilde{\alpha},\tilde{\theta})}{\partial(x,y,z)}.$$

From these identeties it foolows that

$$u = u(x,y)\left(\tilde{u} = \tilde{u}(x,y)\right) \Rightarrow \theta \equiv \frac{\pi}{2}\left(\tilde{\theta} \equiv \frac{\pi}{2}\right);\ u = u(\alpha,\theta)\left(\tilde{u} = \tilde{u}(\alpha,\theta)\right);$$

$$\upsilon = \upsilon(\alpha,\theta)\left(\tilde{\upsilon} = \tilde{\upsilon}(\alpha,\theta)\right);\quad u_z = \varphi(u_x,u_y)\left(\tilde{u}_z = \tilde{\varphi}(\tilde{u}_x,\tilde{u}_y)\right), \text{то } \operatorname{div}\mathbf{G} = 0\left(\operatorname{div}\tilde{\mathbf{G}} = 0\right).$$

In the planar case we get $\mathbf{v} = \mathbf{v}(x,y,t) = \upsilon\boldsymbol{\tau}, \tilde{\mathbf{v}} = \tilde{\mathbf{v}}(x,y,t) = \tilde{\upsilon}\tilde{\boldsymbol{\tau}},\ \boldsymbol{\tau} = \cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}$, $\tilde{\boldsymbol{\tau}} = \cos\tilde{\alpha}\,\mathbf{i} + \sin\tilde{\alpha}\,\mathbf{j}$, where $\alpha = \alpha(x,y,t)$ and $\tilde{\alpha} = \tilde{\alpha}(x,y,t)$ are the slope of vector lines of the field $\mathbf{v}(\tilde{\mathbf{v}})$ as $t = const$. For incompressible media we have $\operatorname{div}\mathbf{v} = 0, \operatorname{div}\tilde{\mathbf{v}} = 0$, $\mathbf{v} = u_y\mathbf{i} - u_x\mathbf{j} = \operatorname{rot}(u\mathbf{k}), \tilde{\mathbf{v}} = \tilde{u}_y\mathbf{i} - \tilde{u}_x\mathbf{j} = \operatorname{rot}(\tilde{u}\mathbf{k}), \upsilon^2 = u_x^2 + u_y^2, \tilde{\upsilon}^2 = \tilde{u}_x^2 + \tilde{u}_y^2$, where $u = u(x,y,t)$ and $\tilde{u} = \tilde{u}(x,y,t)$ are the flow functions.

From equations (13), (15), and theorem 3 it follows

Theorem 5. The system of two-speed hydrodynamics equations with one pressure (10), (11) for a planar motion $\mathbf{v} = \mathbf{v}(x,y,z,t), \tilde{\mathbf{v}} = \tilde{\mathbf{v}}(x,y,z,t), \upsilon \neq 0,\ \tilde{\upsilon} \neq 0$ can be represented in the forms

$$\mathbf{G} = \operatorname{rot}\left(\alpha(x,y,t)\mathbf{k}\right),\quad \tilde{\mathbf{G}} = \operatorname{rot}\left(\tilde{\alpha}(x,y,t)\mathbf{k}\right) \Rightarrow \operatorname{div}\mathbf{G} = 0,\quad \operatorname{div}\tilde{\mathbf{G}} = 0,$$

$$\operatorname{rot}\mathbf{G} = -(\Delta\alpha)\mathbf{k},\ \operatorname{rot}\tilde{\mathbf{G}} = -(\Delta\tilde{\alpha})\mathbf{k} \Rightarrow \ln\upsilon = \operatorname{div}\mathbf{Q},\ \Delta\ln\tilde{\upsilon} = \operatorname{div}\tilde{Q}, \qquad (16)$$

$$(\Delta\alpha)\mathbf{k} = -\operatorname{rot}\mathbf{Q},\quad (\Delta\tilde{\alpha})\mathbf{k} = -\operatorname{rot}\tilde{\mathbf{Q}},$$

where the fields $\mathbf{G}, \mathbf{Q}, \tilde{\mathbf{G}}, \tilde{\mathbf{Q}}$ defined in (8), (13), and (15).

From theorem 3we have

Corollary 1. Both in the case of plane irrotational motion $(\mathbf{v} = \Delta u(x,y,t), \tilde{\mathbf{v}} = \Delta\tilde{u}(x,y,t))$ with potentials $u(x,y,t), \tilde{u}(x,y,t) \in C^3(D)$, and in the case of a flat motion of an incompressible two-speed continuum $\left(\mathbf{v} = \operatorname{rot}(u(x,y,t)\mathbf{k}) = u_y\mathbf{i} - u_x\mathbf{j}, \left(\tilde{\mathbf{v}} = \operatorname{rot}(\tilde{u}(x,y,t)\mathbf{k}) = \tilde{u}_y\mathbf{i} - \tilde{u}_x\mathbf{j}\right)\right.$ with a flow function $u(x,y,t), \tilde{u}(x,y,t) \in C^3(D)$ for the quantities $\alpha_x, \alpha_y, \upsilon = |\mathbf{v}|,\ \mathbf{Q}, \mathbf{S}, \mathbf{V} = -\upsilon^2\mathbf{S}$, $\operatorname{div}\mathbf{V}$, $\operatorname{rot}\mathbf{V}\ \left(\tilde{\alpha}_x, \tilde{\alpha}_y, \tilde{\upsilon} = |\tilde{\mathbf{v}}|, \tilde{\mathbf{Q}}, \tilde{\mathbf{S}}, \tilde{\mathbf{V}} = -\tilde{\upsilon}^2\tilde{\mathbf{S}}, \operatorname{div}\tilde{\mathbf{V}}, \operatorname{rot}\tilde{\mathbf{V}}\right)$ we have the same expressions through derivatives "$u(\tilde{u})$"

$$\upsilon = \sqrt{g},\ g = u_x^2 + u_y^2,\ \tilde{\upsilon} = \sqrt{\tilde{g}},\ \tilde{g} = \tilde{u}_x^2 + \tilde{u}_y^2,\quad \mathbf{Q} = \frac{\Delta u \nabla u}{g}, \mathbf{S} = \operatorname{rot}(\alpha\mathbf{k}), \tilde{\mathbf{Q}} = \frac{\Delta\tilde{u}\nabla\tilde{u}}{\tilde{g}}, \tilde{\mathbf{S}} = \operatorname{rot}(\tilde{\alpha}\mathbf{k}),$$

$$\mathbf{V} = \frac{1}{2}\nabla\left(u_x^2 + u_y^2\right) - \Delta u \nabla u = -\left(u_x^2 + u_y^2\right)\mathrm{rot}\left(\alpha \mathbf{k}\right) =$$
$$= \left(u_y u_{xy} - u_x u_{yy}\right)\mathbf{i} + \left(u_x u_{xy} - u_y u_{xx}\right)\mathbf{j} = \left(\nabla u \times \nabla\right)\nabla u, \quad (17)$$

$$\mathrm{div}\mathbf{V} = 2\left(u_{xy}^2 - u_{xx}u_{yy}\right), \quad \mathbf{rot}\mathbf{V} = -\left\{u_y\left(\Delta u\right)_x - u_x\left(\Delta u\right)_y\right\}\mathbf{k}, \quad (18)$$

$$\tilde{\mathbf{V}} = \frac{1}{2}\nabla\left(\tilde{u}_x^2 + \tilde{u}_y^2\right) - \Delta \tilde{u} \nabla \tilde{u} = -\left(\tilde{u}_x^2 + \tilde{u}_y^2\right)\mathrm{rot}\left(\tilde{\alpha} \mathbf{k}\right) =$$
$$= \left(\tilde{u}_y \tilde{u}_{xy} - \tilde{u}_x \tilde{u}_{yy}\right)\mathbf{i} + \left(\tilde{u}_x \tilde{u}_{xy} - \tilde{u}_y \tilde{u}_{xx}\right)\mathbf{j} = \left(\nabla \tilde{u} \times \nabla\right)\nabla \tilde{u}, \quad (19)$$

$$\mathrm{div}\tilde{\mathbf{V}} = 2\left(\tilde{u}_{xy}^2 - \tilde{u}_{xx}\tilde{u}_{yy}\right), \quad \mathbf{rot}\tilde{\mathbf{V}} = -\left\{\tilde{u}_y\left(\Delta \tilde{u}\right)_x - \tilde{u}_x\left(\Delta \tilde{u}\right)_y\right\}\mathbf{k}, \quad (20)$$

and the followng identities hold $\left(\upsilon \neq 0, \tilde{\upsilon} \neq 0\right)$

$$\mathbf{Q} = \frac{\Delta u \nabla u}{\upsilon^2} = \nabla \ln \upsilon + \mathrm{rot}\left(\alpha \mathbf{k}\right),$$

$$\tilde{\mathbf{Q}} = \frac{\Delta \tilde{u} \nabla \tilde{u}}{\tilde{\upsilon}^2} = \nabla \ln \tilde{\upsilon} + \mathrm{rot}\left(\tilde{\alpha} \mathbf{k}\right) \Leftrightarrow$$

$$\Leftrightarrow \mathbf{R} \stackrel{def}{=} \frac{\Delta u}{\upsilon^2}\mathrm{rot}\left(u\mathbf{k}\right) = -\nabla\alpha + \mathrm{rot}\left(\ln \upsilon \mathbf{k}\right),$$

$$\tilde{\mathbf{R}} \stackrel{def}{=} \frac{\Delta}{\tilde{\upsilon}^2}\mathrm{rot}\left(\tilde{u}\mathbf{k}\right) = -\nabla\alpha + \mathbf{rot}\left(\ln \tilde{\upsilon} \mathbf{k}\right) \Leftrightarrow$$

$$\Delta \ln \upsilon = \mathrm{div}\mathbf{Q}, \qquad \Delta \ln \tilde{\upsilon} = \mathrm{div}\tilde{\mathbf{Q}},$$

$$\left(\Delta\alpha\right)\mathbf{k} = -\mathrm{rot}\mathbf{Q}, \quad \left(\Delta\tilde{\alpha}\right)\mathbf{k} = -\mathrm{rot}\tilde{\mathbf{Q}}.$$

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