

Reliability Analysis of the Sectional Beams Due To Distribution of Shearing Stress

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Abstract: This paper shows the results of the Reliability Analysis of the sectional beams due to distribution of Shear Stress. It is assumed that the load was uniformly distributed over the beam. It is discussed that the distribution of shear stress over the beam. It is discussed that the average shears stress and maximum shear stress across the section of the beam for Weibull distribution. The reliability analysis of distribution of shearing stresses over sectional beams is performed. Also it is derived that the hazard functions for these types of beams. Reliability comparison has also been done for the sectional beams. It is observed that the reliability of the beam decreased when the width (b) of the beam decreases, and the load (F) is high. The reliability of the beam is increased when the height (h) of the triangular section increases, diameter(d) of the circular beam is increased and parameter k decreases

Keyword: Weibull distribution, Shearing Stress, Hazard rate, Reliability analysis, Maximum stress

I. Introduction

There are many situations in which the failure rate cannot be approximated by a straight line. In such cases, the Weibull model can be used to fit the non linear behaviour of Hazard rate $Z(t)$ [1]. Reliability is used for developing the equipment manufacturing and delivery to the user. A reliable system is one which operates according to our expectations. Reliability of a system is the probability that a system perform its intended purpose for a given period of time under stated environment conditions. In some cases system failures occur due to certain type of stresses acting on them. These types of system are called stress dependent models of reliability [2]. These models nowadays studied in many branches of science such as Engineering, Medicine, and Pharmaceutical Industries etc.

In assessing system reliability it is first necessary to define and categorize different modes of system failures. It is difficult to define failure in unambiguous forms. However a system's performance can deteriorate gradually over time and sometimes there is only a fine line between systems success and system failure. Once the system function and failure modes are explicitly stated reliability can be precisely quantified by probability statements.

II. Mathematical And Statistical Model

The probability of failure as a function of time can be defined by

$$F(t) = P(T \leq t), \quad t \geq 0 \quad (1)$$

Where T is a random variable denoting the failure time. **Reliability** function is defined as the probability of success for the intended time t

$$R(t) = 1 - F(t) = P(T > t) \quad (2)$$

The Hazard function $h(t)$ is defined as the limit of the failure rate as the interval approaches zero. Thus the hazard function is the instantaneous failure rates is defined as

$$z(t) = \lim_{\Delta t \rightarrow 0} \frac{f(t)}{R(t)} = \lim_{\Delta t \rightarrow 0} \frac{f(t)}{1 - F(t)} \quad (3) \quad \text{where } f(t) = \frac{dF(t)}{dt}$$

Stress dependent hazard models

Basically, the reliability of an item is defined under stated operating and environmental conditions. This implies that any change in these conditions can effect. The failure rate of almost all components is stress dependent. A component can be influenced by more than one kind of stress. For such cases, a power function model of the form

$$h(t) = z(t)\sigma_1^a \sigma_2^b \quad (4)$$

Where a, b are positive constants, σ_1 and σ_2 are stress ratios for two different kinds of stresses, and $z(t)$ is the failure rate at rated stress conditions.

Weibull Distribution

A Weibull distribution has the density function

$$F(x) = kx^m \exp\left(-\frac{kx^{m+1}}{m+1}\right), x \geq 0$$

Then the distribution function is $F(x) = 1 - \exp\left(-\frac{kx^{m+1}}{m+1}\right), x \geq 0$ (5)

This function consists of two parameters, k and m. These are chosen appropriately.

Hazard Rate

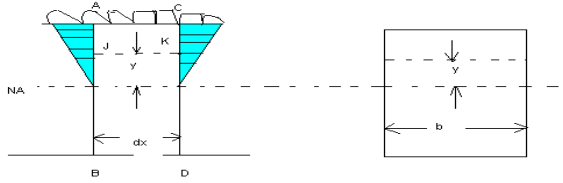
$$z(t) = \frac{f(t)}{1-F(t)}$$

there fore $z(t) = \frac{kt^m \exp\left(-\frac{kt^{m+1}}{m+1}\right)}{1 - \left(1 - \exp\left(-\frac{kt^{m+1}}{m+1}\right)\right)} = kt^m, t \geq 0,$ (6)

III. Shearing Stresses In Beams

Shearing stress at a section in loaded beam

Consider a small portion ABDC of length dx of a beam loaded with uniformly distributed load. We know that when a beam is loaded with a uniformly distributed load, the shear force and bending moment vary at every point along the length of the beam. [3]



Let M = bending moment at AB, $M+dM$ = bending moment at CD,

F = shear force at AB, $F+dF$ = moment of inertia of the section about its neutral axis.

Now consider an elementary strip at a distance y from the neutral axis. Now let σ = intensity of bending stress across AB at distance y from the neutral axis and a = cross sectional area of the strip [4]

We have $\frac{M}{I} = \frac{\sigma}{y}$ or $\sigma = \frac{M}{I} \times y$ similarly, $\sigma+d\sigma = \frac{M+dM}{I} \times y$ Where $\sigma+d\sigma$ = intensive of bending

stress across CD. We know that the force acting across AB = stress \times area = $\sigma \times a = \frac{M}{I} \times y \times a$

Similarly, force acting across CD = $\frac{M+dM}{I} \times y \times a - \frac{M}{I} \times y \times a = \frac{dM}{I} \times y \times a$

The total unbalanced force (F) above the neutral axis may be found by integrating the above

Equation between 0 and $d/2$

$$= \int_0^{d/2} \frac{dM}{I} a \cdot y \cdot dy = \frac{dM}{I} \int_0^{d/2} a \cdot y \cdot dy = \frac{dM}{I} A \bar{y}$$

A = area of the beam above neutral axis and \bar{y} = distance between the centre of gravity of the area and the neutral axis.

We know that the intensity of shear stress $\tau = \frac{\text{total force}}{\text{area}} = \frac{\frac{dM}{I} \cdot A \bar{y}}{dx \cdot b}$ (Where b is the width of the

beam)

$$= \frac{dM}{dx} \times \frac{A \cdot \bar{y}}{Ib} = F \times \frac{A \bar{y}}{Ib} \quad \left(\text{share force } F = \frac{dM}{dx}\right) \quad (7)$$

Distribution of Shearing Stress

We shall discuss the distribution of shear stress over the following sections:

1. Rectangular sections
2. Triangular sections
3. Circular sections

Distribution of shearing stress over a Rectangular section.

Consider a beam of rectangular section ABCD of width and depth. We know that the shear stress on a layer JK of beam, at a distance y from the neutral axis [5],

$$\tau = F \times \frac{A \bar{y}}{Ib} \quad (8)$$

Where F = shear force at the section, A = Area of section above y [shaded area AJKD]

\bar{y} =Distance of the shaded area about the neutral axis, I = Moment of inertia of the whole section about its neutral axis,

$A\bar{y}$ = Moment of the shaded area about the neutral axis, b = width of the section

We know that area of the shaded portion AKLD, $A= b(d/2-y)$

$$\bar{y}= y+\frac{1}{2}\left(\frac{d}{2}-y\right) = y+\frac{d}{4}-\frac{y}{2} = \frac{y}{2}+\frac{d}{4} = \frac{1}{2}\left(y+\frac{d}{2}\right)$$

Substitution the above values of A and \bar{y} in equation (7)

$$\tau = A \times \frac{A \bar{y}}{Ib} = F \times \frac{b\left(\frac{d}{2}-y\right) \times \frac{1}{2}\left(y+\frac{d}{2}\right)}{Ib} = \frac{F}{2I}\left(\frac{d^2}{4}-y^2\right) \quad (9)$$

We see, from the above equation that τ increases as y decreases. At a point, where $y=d/2$, $\tau = 0$ and where y is zero, τ is maximum, we also see that the variation of τ with respect to y is a parabola.

At neutral axis, the value of τ is maximum [6], Thus substituting $y=0$ and $I=\frac{bd^3}{12}$ in the above equation,

$$\tau_{max} = \frac{F}{2 \times \frac{bd^3}{12}} \left(\frac{d^2}{4}\right) = \frac{3F}{2bd} = 1.5\tau_{av} \quad \left(\tau_{av} = \frac{F}{Area} = \frac{F}{bd} \right)$$

(10)

Reliability Function at Maximum Shear Stress of the rectangular section

$$h(t) = z(t) \times \tau_{max} \text{ where shear stress} = \tau_{max} = 1.5 \times \tau_{av}, \quad \tau_{av} = \frac{F}{bd}$$

$$\begin{aligned} \text{Therefore } R(t) &= \exp\left[-\int_0^t h(t)dt\right] = \exp\left[-\int_0^t (z(t) \times \tau_{max}) dt\right] \\ &= \exp\left[-\int_0^t kt^m \times \tau_{max} dt\right] = \exp\left[-k \times \tau_{max} \times \int_0^t t^m dt\right] = \exp\left[-k \times \tau_{max} \times \frac{t^{m+1}}{m+1}\right] \end{aligned} \quad (11)$$

Example: A wooden beam 100mm wide, 250 mm deep and 3m long is carrying a uniformly distributed load of 40kN/m, Determine the maximum shear stress and reliability of shear stress.

Solution: Given width $b=100m$; Depth $d= 250mm$; span $l= 3m= 10^3mm$ and load $w=40kN/m=40N/mm$.

$$F = \frac{wl}{2} = \frac{40 \times (3 \times 10^3)}{2} = 60 \times 10^3 N \text{ and } A = b \cdot d = 100 \times 250 = 25000 mm^2$$

$$\tau_{av} = \frac{F}{A} = \frac{60 \times 10^3}{25000} = 2.4 N/mm^2 \text{ and maximum shear stress, } \tau_{max} = 1.5 \times \tau_{av} = 1.5 \times 2.4 = 3.6 MPa$$

$$\text{Reliability: } R(t) = \exp\left[-k \times \tau_{max} \times \frac{t^{m+1}}{m+1}\right] = 0.5020 \text{ when } k=0.1, m=0.2 \text{ and } t=2$$

Reliability results for Distribution of shearing stress over a Rectangular section

In Table 1, load is varying and other parameters are keeping constant, it is observed that the failure due to shearing stress is increasing when load is increasing as shown in fig 1. In Table -2, width of the beam is increasing and other parameters are keeping constant then the reliability of the beam is increasing as shown in fig 2. In table 3, depth of the beam is increasing it is observed that failure is decreasing as shown in fig 3. In table 4, parameter k is varying and other parameters are constant then it is observed that reliability is decreasing when k is increasing as shown in fig 4. In Table 5, it is observed that failure of the beam caused when time t increasing as shown in fig 5

Table-1				
when $k=0.1, m=0.1, t=2, d=250, b=100$				
F	A=b*d	τ_{av}	τ_{max}	R(t)
60000	25000	2.4	3.6	0.4958
30000	25000	1.2	1.8	0.7042
20000	25000	0.8	1.2	0.7915
10000	25000	0.4	0.6	0.8897
5000	25000	0.2	0.3	0.9432
2000	25000	0.08	0.12	0.9769
1000	25000	0.04	0.06	0.9884
500	25000	0.02	0.03	0.9942
200	25000	0.008	0.012	0.9977
100	25000	0.004	0.006	0.9988

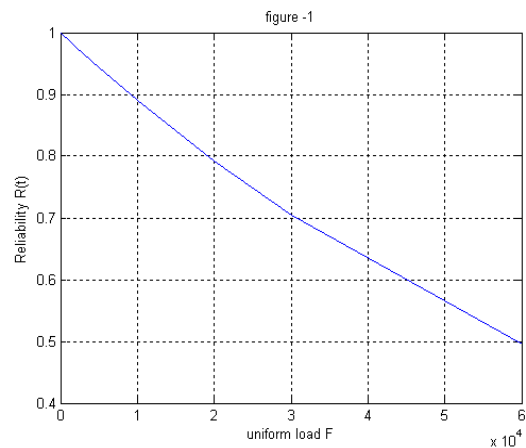


Table-2
when k=0.5, m=0.5 t=3, d=250, F=1000

B	A=b*d	τ_{av}	τ_{max}	R(t)
200	50000	0.02	0.03	0.9494
180	45000	0.0222222	0.0333333	0.9439
160	40000	0.025	0.0375	0.9371
140	35000	0.0285714	0.042857	0.9285
120	30000	0.0333333	0.05	0.9170
100	25000	0.04	0.06	0.9013
80	20000	0.05	0.075	0.8782
60	15000	0.0666667	0.1	0.8410
40	10000	0.1	0.15	0.7712
20	5000	0.2	0.3	0.5947

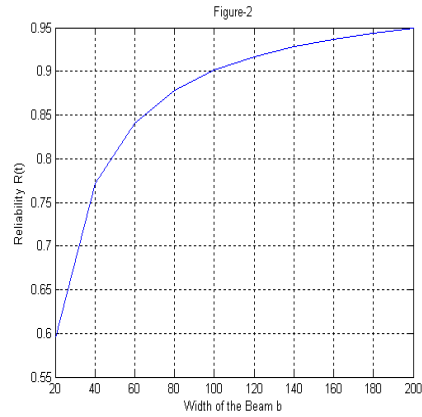


Table-3
when k=0.5, m=0.5 t=5, b=200, F=1000

D	A=b*d	τ_{av}	τ_{max}	R(t)
500	1E+05	0.01	0.015	0.9456
450	90000	0.011	0.017	0.9386
400	80000	0.013	0.019	0.9316
350	70000	0.014	0.021	0.9247
300	60000	0.017	0.025	0.9110
250	50000	0.02	0.03	0.8942
200	40000	0.025	0.038	0.8680
150	30000	0.033	0.05	0.8300
100	20000	0.05	0.075	0.7562
50	10000	0.1	0.15	0.5718

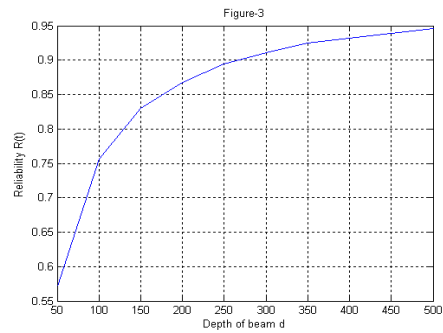
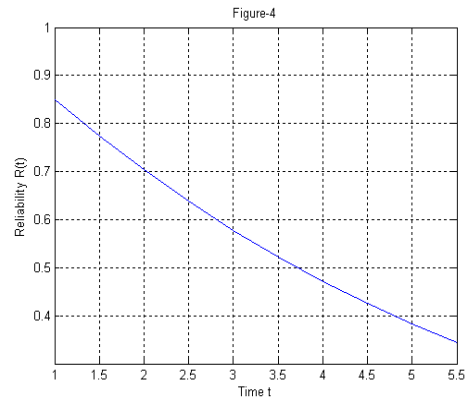


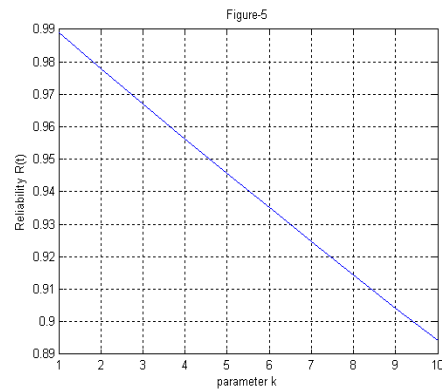
Table-4
when t=5, m=0.5 d=250, b=100, F=60000

k	A=b*d	τ_{av}	τ_{max}	R(t)
1	100000	0.01	0.015	0.9889
2	100000	0.01	0.015	0.9779
3	100000	0.01	0.015	0.9670
4	100000	0.01	0.015	0.9563
5	100000	0.01	0.015	0.9456
6	100000	0.01	0.015	0.9351
7	100000	0.01	0.015	0.9247
8	100000	0.01	0.015	0.9144
9	100000	0.01	0.015	0.9043
1	100000	0.01	0.015	0.8942



Tabel-5
when k=0.05, m=0.1 d=250, b=100, F=60000

t	A=b*d	τ_{av}	τ_{max}	R(t)
1	25000	2.4	3.6	0.8491
1.5	25000	2.4	3.6	0.7744
2	25000	2.4	3.6	0.7042
2.5	25000	2.4	3.6	0.6387
3	25000	2.4	3.6	0.5782
3.5	25000	2.4	3.6	0.5225
4	25000	2.4	3.6	0.4715
4.5	25000	2.4	3.6	0.4249
5	25000	2.4	3.6	0.3825
5.5	25000	2.4	3.6	0.3439



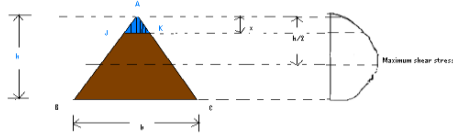
Distribution of shearing stress over a Triangular section

Consider a beam of triangular cross section ABC of base b and height h as shown in Fig. We know that the shear stress on a layer JK at a distance y from the neutral axis. $\tau = F \times \frac{A y}{Ib}$ (12)

F= shear force at the section; $A\bar{y}$ =Moment of the shaded area about the neutral axis

and

I= Moment of the inertia of the triangular section about its neutral axis.



We know that width of the strip JK ,

$$b = \frac{bx}{h}$$

$$\text{Area of the shaded portion AJK. } A = \frac{1}{2}JK \times x = \frac{1}{2}\left(\frac{bx}{h} \times x\right) = \frac{bx^2}{2h}, \quad \bar{y} = \frac{2h}{3} - \frac{2x}{3} = \frac{2}{3}(h - x)$$

$$\text{and substituting the values of b, A and } \bar{y} \text{ in equation (12) } \tau = F \times \frac{\frac{bx^2}{2h} \times \frac{2}{3}(h-x)}{I \times \frac{bx}{h}} = \frac{F}{3I} \times [x(h-x)] = \frac{F}{3I} \times (hx - x^2) \quad (13)$$

Thus we see that the variation of τ with respect to x is parabola. We also see that as a point where $x=0$ or $x=h$, $\tau = 0$. At neutral axis, where $x = \frac{2h}{3}$,

$$\tau = \frac{F}{3I} \left[h \times \frac{2h}{3} - \left(\frac{2h}{3}\right)^2 \right] = \frac{F}{3I} \times \frac{2h^2}{9} = \frac{2Fh^2}{27I} = \frac{2Fh^2}{27 \times \frac{bh^3}{36}} = \frac{8F}{3bh} = \frac{4}{3} \times \frac{F}{AREA} = 1.33\tau_{av} \quad \left(\text{AREA} = \frac{bh}{2} \right)$$

(14)

Now for maximum intensity, differentiating the (13) and equating to zero,

$$\frac{d}{dx} \left[\frac{F}{3I} (hx - x^2) \right] = 0 \quad \text{Therefore} \quad h - 2x = 0 \quad \text{or} \quad x = \frac{h}{2}$$

now substituting this value of x in equation (13)

$$\tau_{max} = \frac{F}{3I} \left[h \times \frac{h}{2} - \left(\frac{h}{2}\right)^2 \right] = \frac{Fh^2}{12I} = \frac{Fh^2}{12 \times \frac{bh^3}{36}} = \frac{3F}{bh} = \frac{3}{2} \times \frac{F}{Area} = 1.5\tau_{av} \quad \left(I = \frac{bh^3}{36} \right) \quad (15)$$

Reliability Function at Maximum Shear Stress of the triangular section

$$h(t) = z(t) \times \tau_{max} \quad \text{where shear stress} = \tau_{max} = 1.5 \times \tau_{av}, \quad \tau_{av} = \frac{F}{bh}$$

$$\text{Therefore } R(t) = \exp \left[- \int_0^t h(t) dt \right] = \exp \left[- \int_0^t (z(t) \times \tau_{max}) dt \right]$$

$$= \exp \left[- \int_0^t kt^m \times \tau_{max} dt \right] = \exp \left[-k \times \tau_{max} \times \int_0^t t^m dt \right] = \exp \left[-k \times \tau_{max} \times \frac{t^{m+1}}{m+1} \right] \quad (16)$$

Example: A beam of triangular cross section having base width of 100 mm and height of 150mm is subjected to a shear force of 13.5 kN. Find the value of maximum shear stress and sketch the shear stress distribution along the depth of beam.

Solution: Given base width (b) = 100mm; height (h) = 150mm and shear force F = 13.5kN = 13.5 × 10³N

$$A = \frac{bh}{2} = \frac{100 \times 150}{2} = 7500 \text{ mm}^2$$

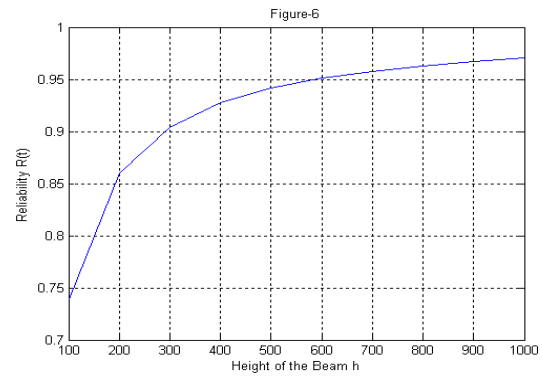
$$\tau_{av} = \frac{F}{A} = \frac{13.5 \times 10^3}{7500} = \frac{1.8 \text{ N}}{\text{mm}^2} = 1.8 \text{ MPa} \quad \text{And maximum shear stress} \quad \tau_{max} = 1.5 \times \tau_{av} = 1.5 \times 1.8 = 2.7 \text{ MPa}$$

$$\text{Reliability: } R(t) = \exp \left(-k \times \tau_{max} \times \frac{t^{m+1}}{m+1} \right) = 0.9504 \quad \text{when } k=0.01, \quad m=0.5 \quad \text{and } t=2$$

Reliability results for Distribution of shearing stress over a Triangular section

In Table-6, height of the triangular section is keep changing and other parameters are keeping constant, it is observed that the reliability of the beam of type triangular section is increasing when height of the triangular section is increasing

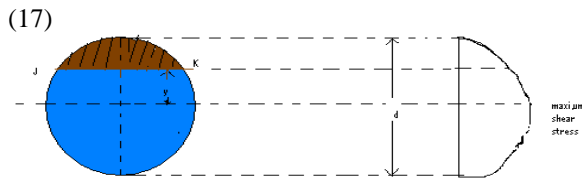
h	A	τ_{av}	τ_{max}	R(t)
100	5000	2.7	4.05	0.7394
200	10000	1.35	2.025	0.8599
300	15000	0.9	1.35	0.9043
400	20000	0.675	1.0125	0.9273
500	25000	0.54	0.81	0.9414
600	30000	0.45	0.675	0.9509
700	35000	0.385714	0.578571	0.9578
800	40000	0.3375	0.50625	0.9630
900	45000	0.3	0.45	0.9670
1000	50000	0.27	0.405	0.9703



Distribution Of Shearing Stress Over A Circular Section

Consider a circular section of diameter d as shown in fig. We know that the shear stress on a layer JK at a distance y from the neutral axis,

$$\tau = F \times \frac{A \bar{y}}{Ib}$$



F=Shear force at the section, $A\bar{y}$ = Moment of the shaded area about the neutral axis,
 r= Radius of the circular section, I= Moment of inertia of the circular section and b= width of the strip JK
 We know that in a circular section Width of the strip JK, $b=2\sqrt{r^2 - x^2}$ and area of the shade strip,
 $A= 2\sqrt{r^2 - x^2}.dy$ Moment of this area about the neutral axis $= 2y\sqrt{r^2 - x^2}.dy$
 Now moment of the whole shaded area about the neutral axis may be found out by integrating the above equation between the limits y and r, i.e.,

$$A\bar{y} = \int_y^r 2y\sqrt{r^2 - x^2}.dy = \int_y^r b.y.dy$$

(18)

We know that width of the strip Jk, $b=2\sqrt{r^2 - x^2}$ therefore $b^2 = 4(r^2 - x^2)$
 Differentiation both sides of the above equation. $2b.db = 4(-2y) dy \therefore y.dy = -b.db/4$
 Substituting the value of y.dy in equation (18), we get $A\bar{y} = \int_y^r b(-b.db/4) = -\frac{1}{4} \int_y^r b^2.db$

(19)

We know that when $y=0$, width $b=b$ and when $y=r$, width $b=0$. Therefore, the limits of integration may be changed from y to r, from b to zero in equation (19) $A\bar{y} = -\frac{1}{4} \int_b^0 b^2.db = \frac{1}{4} \int_0^b b^2.db = \frac{b^3}{12}$
 Now substitute this value of $A\bar{y}$ in our original formula for the shear stress, i.e.,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{\frac{b^3}{12}}{Ib} = F \times \frac{b^2}{12I} = F \times \left[\frac{(2\sqrt{r^2 - x^2})^2}{12I} \right] = F \times \frac{r^2 - x^2}{3I}$$

(20)

Thus we again see that τ increases as y decreases, At a point where $y=r$, $\tau=0$, and where y is zero, τ is maximum, we also see that the variation of τ with respect to y is a parabolic curve. We see that at neutral axis τ is maximum.

Substituting $y=0$ and $I = \frac{\pi}{64} \times d^4$ in the above equation. $\tau_{max} = F \times \frac{r^2}{3 \times I} = F \times \frac{(\frac{d}{2})^2}{3 \times \frac{\pi}{64} \times d^4} = \frac{4F}{3 \times \frac{\pi}{4} \times d^2} = 1.33\tau_{av}$

(21)

Reliability Function at Maximum Shear Stress of the circular section

$h(t) = z(t) \times \tau_{max}$ where shear stress $= \tau_{max} = 1.33\tau_{av}$, $\tau_{av} = \frac{F}{A}$, $A = \frac{\pi}{4} \times d^2$

Therefore $R(t) = \exp \left[- \int_0^t h(t) dt \right] = \exp \left[- \int_0^t (z(t) \times \tau_{max}) dt \right]$

$$= \exp \left[- \int_0^t k t^m \times \tau_{max} dt \right] = \exp \left[-k \times \tau_{max} \times \int_0^t t^m dt \right] = \exp \left(-k \times \tau_{max} \times \frac{t^{m+1}}{m+1} \right) \quad (22)$$

Example: A circular beam of 100mm diameter is subjected to a shear force of 30kN. Calculate the value of maximum shear stress and sketch the variation of shear stress along the depth of the beam.

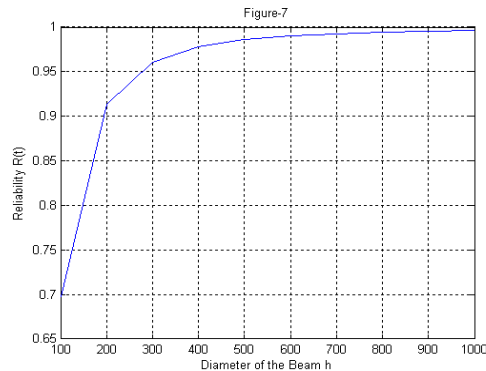
Solution: Given: Diameter $d= 100\text{mm}$ and shear force $f= 30\text{kN}=30 \times 10^3\text{N}$, $A= \frac{\pi}{4}(d^2) = \frac{\pi}{4}(100^2) = 7854\text{mm}^2$, $\tau_{av} = \frac{F}{A} = 3.82\text{N/mm}^2=3.82\text{MPa}$ and maximum shear stress, $\tau_{max} = 1.33 \times \tau_{av} = 1.33 \times 3.82 = 5.08\text{MPa}$

Reliability: $R(t) = \exp \left(-k \times \tau_{max} \times \frac{t^{m+1}}{m+1} \right) = 0.9087$ when $k=0.01$, $m=0.5$ and $t=2$

Reliability results for Distribution of shearing stress over a circular section

In Table -7, diameter of the circular section is changing and other parameters are keeping constants then the reliability of the beam of type circular section is increasing when diameter of the circular section is increasing.

d	A	τ_{av}	τ_{max}	R(t)
100	7857.143	1.272727	1.909091	0.6977
200	31428.57	0.318182	0.477273	0.9139
300	70714.28	0.141414	0.212121	0.9608
400	125714.3	0.079545	0.119318	0.9778
500	196428.6	0.050909	0.076364	0.9857
600	282857.1	0.035354	0.05303	0.9901
700	385000	0.025974	0.038961	0.9927
800	502857.1	0.019886	0.02983	0.9944
900	636428.6	0.015713	0.023569	0.9956
1000	785714.3	0.012727	0.019091	0.9964



IV. Conclusion

Reliability analysis of beams was obtained for constant failure rate by estimating the reliability levels at various values of width (b); depth (d); height (h); diameter (d).

- Generally, the width (b) of the beam decreases, the reliability of the rectangular section of the beam decreases when the depth of the beam kept constant.
- Also, the height of the triangular section increases, the reliability of the triangular section of the beam increases when the width of the beam kept constant.
- Also, as diameter of the circular beam is increased, the reliability of the beam is increased.
- It could also be seen that at higher load (F), the reliability of the beam decreases.
- When the hazard rate is increases, the reliability of the beam is decreased.

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