

## Analysis of the Use of Universal Distribution Factors in SEC Power Grid

A. Alodhaiani<sup>1,\*</sup>, Y. Alturki<sup>2</sup>, M. El-Kady<sup>2</sup>

<sup>1</sup> Saudi Electricity Company (SEC), Riyadh, Saudi Arabia

<sup>2</sup> Department of Electrical Engineering, SEC Chair for Reliability and Security, King Saud University, Riyadh, Saudi Arabia

---

**Abstract:** Distribution factors have been extensively used in many power system analysis and planning studies. In recent power system studies, the AC distribution factors are insensitive to the operating point and relatively sensitive at certain degree to changes in network topology. These factors are linear approximations of sensitivities of variables with various inputs. This paper presents the calculation of the universal distribution factors (UDF's) applies them on several practical scenarios of Saudi Electricity Company (SEC) power grid. The results are analyzed and evaluated considering various system conditions of SEC load. The results show that the accuracy of the used approach is acceptable compared with exact method. This is practically beneficial to SEC in computing its grid complex power flows using UDF's at the base case without the need to recalculate UDF's which save efforts and time.

**Key words:** Distribution Factors, Sensitivity Factors, Power System, Electricity Market.

---

### I. Introduction

AC distribution factors and its application play an important role in operation and control of power systems and have been addressed extensively in the literature. In this respect, considerable research efforts have been expended during recent years and it's going on in developing theories, new techniques, policies and required criteria for distribution factors and its application in operation and control of power system. A novel model for evaluating universal distribution factors that is suitable for an extensive range of power system pricing studies and power system planning and analysis were introduced in [1]. The author present the computation of the universal distribution factors in addition to their sensitivities in with respect to the voltage profile of line. The availability and utilization of the existing transmission systems and the operation of the competitive markets in electricity system are very much affected by the congestion. The distribution factors are linear approximations of sensitivities of variables with respect to various inputs and are computed for a specified network topology and parameter values. Hence, they are very important in designing the congestion model for various market applications. The authors in reference [2] investigated the analytical characteristics, the robustness and the quality of the approximations provided the power transfer distribution factors (PTDFs). These factors have provided a reliable approximation for large power system networks. The errors of the approximations are within the permissible for a broad range of conditions including contingencies. The concepts and the development of the AC distribution factors in power systems has been addressed in the literature [3-5]. In this regard, the authors of reference [3] presented a solution algorithm to improve the poor accuracy of generalized generation shift distribution factor (GGDF). A modified power transfer distribution factor (MPTDF) was used to consider the nonconformity of the demand change. When the system operating point is shifted to a new one, the power flows in the line can be got directly without running a load flow program. By means of four example systems, the potential behavior of the proposed method was demonstrated with very fast execution time and with high accuracy compared with the fast-decoupled load flow (FDLF) technique. On the other hand, the authors of reference [4] has analyzed the distribution of source power from the energy source, by deriving and defining the complex power dynamics distribution factors based on the circuit theory. The form of these factors shows that the power distribution is not only related to the network topology and circuit parameters, but also related to the dynamic state of the system. The source powers are linear distribution in the network according to these factors and whereas the branch flows and losses in the network are different forms of source powers. The example demonstrated that the results calculated by proposed method are unique and they satisfy complex power conservation. By using the Distribution factors, the contributions of the transmission users towards line flows in an electricity market are identified and the transmission charges are fairly fixed. However, due to some limitations, the Distribution factors cannot be applied for the wider applications of electricity market practices. In reference [5], a new technique was proposed to evaluate the transmission line flows more accurately. The new justified distribution factors approach has shown better result than other commonly used distribution factors when tested in a realistic electricity.

Power transfer distribution factors depend on the operating point and topology of an electric power system. However, in recent researches, the power transfer distribution factors were shown theoretically to be relatively insensitive to the operating point. The authors of reference [6] provided empirical corroboration of this theoretical result. The several researches and test conducted on the three principal interconnections namely in North America, the Eastern interconnection, the Western interconnection, and the Electric Reliability Council of Texas (ERCOT) interconnection has shown consistent observations. On the other hand, the authors of reference [7], have studied the Power Transfer Distribution Factors (PTDF) and the underlying flow-based model proposed for capacity determination and allocation in electricity grids. Several factors such as topological and seasonal changes as well as zone building which influence PTDF coefficients are critically analyzed through AC power flow simulation of the Union for the Co-ordination of Transmission of Electricity (UCTE) transmission network (transmission system operators in continental Europe). Finding of border capacities are also analyzed. Also, the author of reference [8] proposed a methodology to create a family of zonal Power Transfer Distribution Factor (PTDF) matrices which can be used to find the power flows accurately for the Zonal market under dynamic condition. Introduction of data mining methods for selected typical hours based on area balance and PTDF matrices of these typical hours calculated to form a family are tested on the New England system. It is observed that the error, when using a family of PTDF matrices for estimating cross-border power flows in a zonal market model is very much less than the method of using only one PTDF matrix.

A model for calculating distribution factors were applied on a several scenarios of SEC power grid in this paper. We analyzed the calculation of UDF's on the SEC grid at all scenarios considering various percentages of the SEC load. The evaluation of the UDF's with respect the operating point and network topology were also discussed.

## II. Universal Distribution Factors (UDF's)

### 1.1. UDF's formula

$N_b = N + N_d$  where,  $N_b$  is the number of the buses in the power system with  $N$  and  $N_d$  is the number of the generation buses and load buses in the power system respectively. Also,  $N_m$  is the number of the transmission branches. We can calculate line complex power flows ( $S_{pq}$ ) between bus  $p$  and bus  $q$  from follow equation;

$$S_{pq}^* = V_p^* I_{pq} \quad (1)$$

Where  $S_{pq}$ ,  $V_p$  and  $I_{pq}$  denote the line complex power flows of  $pq$  line at bus  $p$ , voltage at node  $p$ , and  $p = 1, 2, \dots, N$  and current in line  $pq$ , respectively.

The complex power flow between buses  $p$  and  $q$  on line  $m$  can be expressed as;

$$S_p^* = [l V_p^* + (1-l) V_q^*] I_m = \varphi_m I_m \quad (2)$$

Where  $l$  denote the location on line  $m$  at which  $S_m$  is evaluated. For illustration, if  $l = 0$  this illustrate that  $S_m$  is evaluated at bus  $q$ , if  $l = 1$  this illustrate that  $S_m$  is evaluated at bus  $p$  and if  $l = 0.5$  this illustrate that  $S_m$  is evaluated at the mid-point on line  $m$ , that is;

$$S_p^* = [(V_p^* + V_q^*)/2] I_m \quad (3)$$

Where;  $I_m$  is the current on line  $m$ .

The bus admittance matrix  $Y$  of the grid can be determined as follow;

$$V = ZI \quad \text{or} \quad V = Y^{-1}I \quad (4)$$

Diagonal matrix  $\varphi$  can be written as;

$$\varphi_m = \text{diagonal} \{ \varphi_1, \varphi_2, \dots, \varphi_{n_m} \}$$

The  $n_m$ -vector of complex conjugate line powers can be written as follows;

$$S_m^* = \varphi_m I_m \quad (5)$$

Where  $S_m^*$  and  $I_m$  denotes the  $n_m$  vector of complex conjugate line powers  $\{S_m\}$  and  $n_m$  vector of complex line currents  $\{I_m\}$ , respectively.

The bus incidence matrix that is A branch-to-node incidence matrix where  $A = (n_m \times n_b)$ , which consist of one row for each branch and one column for each bus with an entry 0, 1 or -1. Where,  $A_{bm} = 0$  if the branch  $m$  is not connected to the bus  $b$ ,  $A_{bm} = -1$  if the current in branch  $m$  is directed toward bus  $b$  and  $A_{bm} = 1$  if the current in branch  $m$  is directed away from bus  $b$ .

Therefore;

$$V_m = A V \quad (6)$$

Hence,

$$I_m = Y^P V_m = Y^P A (Y^{-1} I) \quad (7)$$

Where,  $Y^P$  primitive admittance matrix of two coupled branches. The diagonal elements of the primitive admittance matrix represent line self-admittances and off-diagonal elements represent mutual line admittances.

Hence;

$$\varphi_m I_m = \varphi_m (Y^P A Y^{-1}) I \quad (8)$$

The bus voltage matrix can be written as;

$$E = \text{diagonal} \{V_1, V_2, \dots, V_n\}$$

Where  $E$  is the diagonal matrix of bus voltages. The  $n$ -vector  $S^*$  of complex conjugate bus powers can be written as follows;

$$S^* = E^* I \quad (9)$$

or

$$I = E^{*-1} S^* \quad (10)$$

Therefore;

$$S_m^* = \varphi_m I_m = [\varphi_m (Y^P A Y^{-1}) E^{*-1}] S^* = UDF S^* \quad (11)$$

The universal distribution factors matrix  $UDF$  relates line complex power flows  $S_m$  to the bus injected complex powers  $S_b$ , with  $n_b$  and  $n_m$  denoting, respectively, number of buses and number of lines in the system, as follows;

$$S_m = UDF S \quad (12)$$

Where is the universal distribution factors matrix  $UDF$  is;

$$UDF = \varphi_m^* (Y^{P*} A Y^{*-1}) E^{-1} \quad (13)$$

### 1.2. UDF's Evaluation

To evaluate the validity and the effectiveness of the computed  $UDF$ 's, the system is analyzed and studied with applying a number of scenarios. These scenarios includes a change demands and generation. Line complex power flows are calculated as in equation (14) in different scenarios.

$$P_{L\ new} = UDF P_{B\ new} \quad (14)$$

Where  $P_{L\ new}$  is the new line complex power flow after change the bus injected power ( $P_{B\ new}$ ),

$$P_{L\ new} = P_{L_0} + \Delta P_L \quad (15)$$

and

$$P_{B\ new} = P_{B_0} + \Delta P_B \quad (16)$$

Which  $\Delta P_L$  and  $\Delta P_B$  denotes the changes in the lines complex power flows and the changes in bus injected powers respectively.

## II. Case Studies

In this section, the universal distribution factors ( $UDF$ 's) model proposed in [1] is applied on the 2012 real system of Saudi Electricity Company (SEC). SEC grid consists of 2592 buses, 711 generation units, 2251 transmission lines, 1898 transformers, 35 line shunts, 248 switched shunts and 139 tie-lines. Figure 1 shows the consumption of electricity in Kingdom of Saudi Arabia (KSA) which is totally covered by SEC interconnected network (generation, transmission and distribution). The electricity peak load of KSA in SEC power grid happened during the summer season at 48138 MW which was recorded in 21 July 2012.

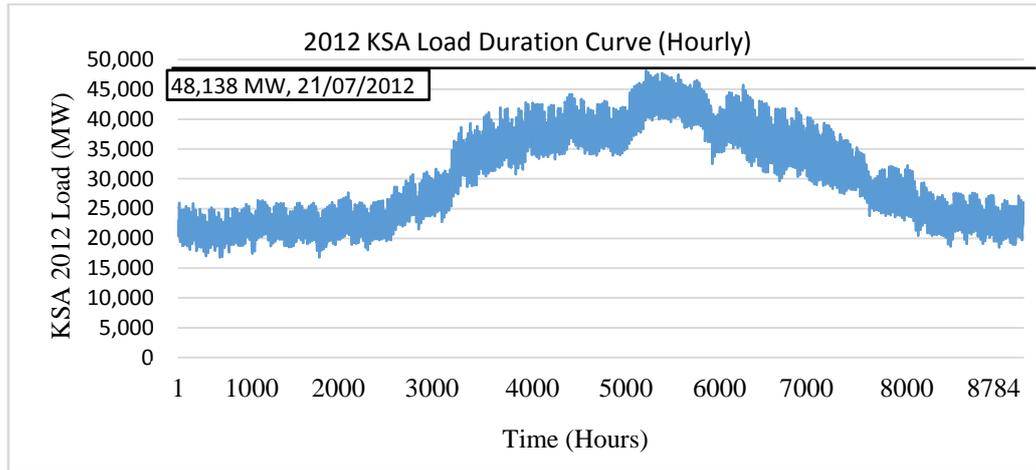


Fig. 1 Consumption of the electricity in KSA

The real system of Saudi Electricity Company grid is reduced to 15 tie lines and 9 buses as shown in Figure 2. The 9 buses of the reduced model denotes four generations owned by SEC, four load areas and one independent power plant owned by Dhurma Electricity Company. The four load areas represent central operating area (COA), west operating area (WOA), east operating area (EOA) and the south operating area (SOA). The 15 tie lines represent the corridors connected all loads and generators. The UDF's have been determined taking into account all the several scenarios applied on the reduced model of real SEC system.

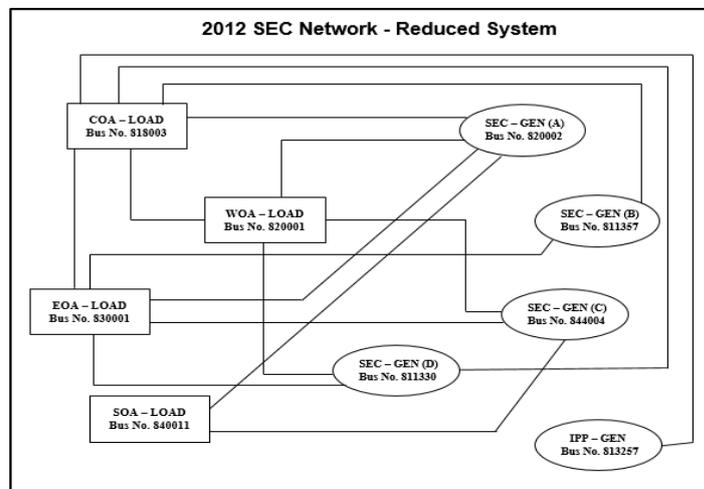


Fig. 2 Reduced model of SEC interconnected system

The UDF's are calculated at the base case, which is the solved base case of 2012 SEC network. Three practical simulated scenarios were conducted to examine the validity and accuracy of using the universal distribution factors of the base case when operating points and network topologies of SEC electric power system change. The results are analyzed and discussed.

Table 1 shows the details of the three scenarios that have been conducted to evaluate the application of universal distribution factors on SEC network.

Table 1. Details of the simulated scenarios for the UDF's evaluations

Scenario no.	Scenario Description
Base case	Solved base case of 2012 with no contingencies
Scenario no. 1	10% loss of generation owned by SEC generation Company SEC – GEN (D) bus No. 811330
Scenario no. 2	10% Increase of demand WOA bus#820001
Scenario no. 3	25% Outage of line between SEC-GEN (A) bus No. 820002 and COA demand bus No. 818003

## IV. Results and Discussion

### 4.1 UDF's analysis at base case

The universal distribution factors matrix is determined at the base case with no contingency cases at parameter  $l = 0.5$  ( $S_m$  is evaluated based on voltages at the mid-point of line  $m$ ). The calculated universal distribution factors of all reduced network lines are shown in Table 2.

Table 2. Distribution factor values for all lines at base case

Line #	From Bus#	To Bus#	Distribution Factor Values at all lines								
			820002	811357	844004	811330	813257	818003	820001	830001	840011
1	811357	818003	0.0197	0.8399	0.0022	0.0046	0.1311	0.1302	0.0162	0.0023	0.0014
2	818003	830001	0.0357	0.2321	0.0041	0.0083	0.2381	0.2365	0.0294	0.0043	0.0026
3	818003	820001	0.0031	0.0143	0.0008	0.0011	0.0147	0.0146	0.0041	0.0008	0.0009
4	820001	844004	0.006	0.0411	0.073	0.0561	0.0415	0.0412	0.0031	0.0715	0.0707
5	820001	820002	0.7291	0.3094	0.0635	0.0156	0.3168	0.3146	0.1952	0.0626	0.0885
6	820001	811330	0.812	0.4965	0.3087	0.23	0.5065	0.5029	0.8816	0.3055	0.3291
7	844004	840011	0.0193	0.0498	0.1037	0.0814	0.0501	0.0498	0.0086	0.1016	0.8626
8	830001	844004	0.0612	0.1124	0.7298	0.1988	0.0146	0.1117	0.077	0.3575	0.011
9	840011	820002	0.0215	0.0469	0.1005	0.0785	0.0473	0.0469	0.0063	0.0984	0.1336
10	818003	811330	0.0472	0.2503	0.0224	0.0096	0.2565	0.2547	0.0396	0.0223	0.0234
11	818003	820002	0.0496	0.1497	0.0104	0.0127	0.1537	0.1526	0.0409	0.0102	0.0119
12	818003	813257	0	0	0	0	0.9788	0	0	0	0
13	830001	820002	0.1345	0.0667	0.0122	0.0333	0.0683	0.0678	0.1108	0.0115	0.0169
14	830001	811357	0.0201	0.1396	0.0023	0.0047	0.1342	0.1333	0.0166	0.0024	0.0015
15	830001	811330	0.3847	0.4978	0.9381	0.0412	0.5021	0.4986	0.344	0.9431	0.921

From line no. 1 in Table 2, it is interesting to notice that the bus no. 811357 has the largest DF value in this line, while the other buses have less influence on this line. In other words, bus 811357 is generation bus and the amount of the transmitted power through this line is very high. DF's values of the buses 844004, 830001 and 840011 are much smaller than the rest of the buses, which indicates that the amount of power injection or withdrawal change in these buses have less influence on the tie-line no. 1 compared with the other buses. From line no. 12 in Table 2, we notice that the bus no. 813257 has DF value while the other buses do not have DF values because of this bus is connected only to one bus in the reduced network and the number of this bus is 818003. The rest lines have the same properties as line no.1.

The UDF's matrix can be used to measure the sensitivity of the power flow to the power variation from the buses. Therefore, we can utilize the UDF's matrix to indicate the load demand in the system. The real and imaginary values of the computed UDF's have positive and negative values as indicated in Appendix A. These values are leading us to two points, the first one is; if the UDF's values positive, then the power flow on the corresponding branch is decreasing, and the second point is; if the UDF's values negative, then the power flow on the corresponding branch is increasing.

### 4.2 UDF's evaluation at all scenarios

The line power flows have been determined by using one universal distribution factors matrix calculated at the base case. These line power flows have been calculated at all the scenarios as shown in Table 3 considering contingency cases as represented in Table 1 and using the UDF's at the base case. The calculated line power flows are compared with the exact line MVA power flows obtained by AC power flow at each line. Table 3 shows the error values between the exact and computed line MVA power flows at all scenarios.

Table 3. Line complex power-flows  $S_p$ -values and error values at all scenario for all lines.

INC-TAG	From Bus #	To Bus #	Line #	Scenario #01			Scenario #02			Scenario #03		
				Line Flow (MVA) Exact	Line Flow (MVA) Calc.	MVA-Error (%)	Line Flow (MVA) Exact	Line Flow (MVA) Calc.	MVA-Error (%)	Line Flow (MVA) Exact	Line Flow (MVA) Calc.	MVA-Error (%)
Pu=1.00	820002	820001	5	9507.70	9436.61	0.748	9957.83	9873.62	0.846	9354.23	9659.58	3.264
Pu=1.00	820002	840011	9	287.68	295.00	2.545	213.25	209.61	1.704	278.66	288.05	3.370
Pu=1.00	820002	818003	11	1397.01	1408.01	0.787	1357.42	1372.31	1.097	1120.33	903.42	19.361
Pu=1.00	820002	830001	13	157.90	162.27	2.767	85.87	87.78	2.223	147.55	151.03	2.362
Pu=1.00	811357	818003	1	9305.88	9269.97	0.386	9336.74	9299.44	0.400	9245.61	9917.95	7.272
Pu=1.00	811357	830001	14	853.24	839.37	1.625	864.36	848.85	1.794	927.02	995.61	7.398
Pu=1.00	844004	820001	4	163.41	160.57	1.733	224.49	220.41	1.816	164.35	167.82	2.115
Pu=1.00	844004	840011	7	3663.82	3665.36	0.042	3749.15	3741.25	0.211	3673.42	3680.78	0.200
Pu=1.00	844004	830001	8	7736.06	7772.33	0.469	7548.03	7538.90	0.121	7426.17	7485.09	0.793
Pu=1.00	811330	820001	6	2816.72	2813.91	0.100	3485.36	3477.38	0.229	2929.26	2981.45	1.782
Pu=1.00	811330	818003	10	1765.84	1739.79	1.475	1794.98	1765.08	1.665	1910.23	1966.53	2.947
Pu=1.00	811330	830001	15	4295.15	4202.38	2.160	4583.56	4504.98	1.714	4782.23	4762.82	0.406
Pu=1.00	813257	818003	12	598.63	585.92	2.123	598.63	585.92	2.123	598.66	595.62	0.507
Pu=1.00	820001	818003	3	110.26	107.76	2.267	106.33	103.40	2.754	119.25	126.07	5.721
Pu=1.00	830001	818003	2	1630.62	1605.35	1.550	1650.86	1622.46	1.721	1758.57	1890.30	7.491

Results show that the absolute value of error percentages between exact and calculated line power flows for all the network lines at scenarios no. 1 and 2 are small and negligible. It is observed that the error values for scenario no. 1 and 2 ranges from 0% to 2.7%. Therefore, the computed UDF's at the base case can be used on SEC network to calculate the line complex power flows for all the network lines. It can be concluded that when system load increases up to 10% or the network generation decreases to the percentage of 10% reduction, SEC can use UDF's of the base case with acceptable accuracy. This is useful to SEC as it saves time and effort by avoiding calculating UDF's and AC powers flows for network changes with these ranges.

On the other hand, simulated results show that the absolute value of error percentages for all the network lines at scenario no.3 are slightly high for some lines and almost are equal to the percentage of the corridors outages for some lines. The error values of this scenario ranges from 0% to about 10% for all lines except for line 11 at which the error reaches 19.36%. . Therefore, the computed UDF's at the base case can be used at certain degree on SEC network to calculate the line complex power flows for some of the network lines, which gives reasonable results except on line no. 11.

It is worth noticing that error percentages for scenarios 1 and 2 are much lower than those of scenario 3. So, one can conclude that change of network topology may lead to more effect on UDF's resulting in much higher errors than the situation when the change is in system load or generation.. The universal distribution factors are more sensitive at certain degree to the network topology than to the operating point.

## V. Conclusion

This paper presented the universal distribution factors UDF's and it applied them on different practical scenarios on SEC power grid. The evaluation of the UDF's on SEC network considering several scenarios including different variations of the demands as well as changes in network topology conducted but due to limited space three scenarios were presented. The UDF's at base case can only be used on a changed network if the difference between their results and the actual calculated line power flows of the new changed network is negligible (i.e. acceptable error), otherwise, UDF's should be recalculated.

In this paper, three scenarios including generation and system demands changes as well as outage of a tie-line of SEC power grid were simulated. Comparison of simulated results between exact and calculated line power flows for all the network lines shows that the absolute value of the error percentages are acceptable for demand or generation changes as represented in Table. 1. While for system topology change such as line outage, the errors are slightly high for some lines and almost equal to the percentage of the corridors for some lines. In conclusion, the calculated UDF's at the base case of SEC grid can be used to calculate the line power flows for all the network lines in several expected and critical scenarios of the SEC power network with acceptable accuracy. This is beneficial to SEC operation as it saves time and effort on real time when such contingencies occur on its real network.

## **VI. Acknowledgment**

This work was supported by Saudi Electricity Company (SEC) through its Chair in Power System Reliability and Security at Department of Electrical Engineering, King Saud University.

## **References**

- [1] M. A. El-Kady, "Universal Distribution Factors for Power System Open-Market Studies", *IET Gener. Transm. Distrib.*, Vol. 5, No. 3, 2011, pp. 356-359.
- [2] M. Liu and G Gross, "Effectiveness of the Distribution Factor Approximations Used in Congestion Modeling", *Proceedings of the 14th Power Systems Computation Conference*, June 24-29, 2002, Sevilla, Spain.
- [3] Y. C. Chang, W. T. Yang and C. C. Liu, "Improvement on GGDF for power system security evaluation", *IEE Proc. Gener. Transm. Distrib.*, Vol. 141, No. 2, March 1994, pp. 85-88.
- [4] A. Dongping, B. Hai and Y. Yihan, "A New Method of Calculating Complex Power Dynamic Distribution factors by Circuit Theory", *Proceeding of IEEE Power and Energy Engineering Conference (APPEEC), Asia-Pacific*, 2010.
- [5] N. H. Radzi , Z. Y. Dong and R. C. Bansal, "Justified distribution factors approach for pool modelling", *IPEC Conference Proceedings*, Singapore, Oct. 2010, pp. 117-1122.
- [6] K. Dixit, R. Baldick and T. J Overbye, "Empirical Analysis of The Variation of Distribution Factors with Loading", *IEEE Power Engineering Society General Meeting*, Vol. 1, June 2005, pp. 221 – 229.
- [7] C. Duthaler, M. Emery, G. Andersson and M. Kurzidem, "Analysis of The Use of PTDF in the UCTE Transmission Grid", 2008.
- [8] D. Huang, "Methodology of Dynamic PTDF for the Zonal Model", *IEEE Power Engineering Society General Meeting*, Vol. 1, June 2005, pp. 221 – 229.