

Choice of Numerical Integration Method for Wind Time History Analysis of Tall Buildings

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ABSTRACT: Wind tunnel tests are being performed routinely around the world for designing tall buildings but the advent of powerful computational tools will make time-history analysis for wind more common in near future. As the duration of wind storms ranges from tens of minutes to hours while earthquake durations are typically less than a three to four minutes, the choice of a time step size (Δt) for wind studies needs to be much larger both to reduce the computational time and to save disk space. As the error in any numerical solution of the equation of motion is dependent on step size (Δt), careful investigations on the choice of numerical integration methods for wind analyses are necessary. From a wide variety of integration methods available, it was decided to investigate three methods that seem appropriate for 3D-time history analysis of tall buildings for wind. These are modal time history analysis, the Hilber-Hughes-Taylor (HHT) method or α -method with $\alpha=0.1$, and the Newmark method with $\beta=0.25$ and $\gamma=0.5$ (i.e., trapezoidal rule). SAP2000, a common structural analysis software tool, and a 64-story structure are used to conduct all the analyses in this paper. A boundary layer wind tunnel (BLWT) pressure time history measured at 120 locations around the building envelope of a similar structure is used for the analyses. Analyses performed with both the HHT and Newmark-method considering P-delta effects show that second order effects have a considerable impact on both displacement and acceleration response. This result shows that it is necessary to account P-delta effect for wind analysis of tall buildings. As the direct integration time history analysis required very large computation times and very large computer physical memory for a wind duration of hours, a modal analysis with reduced stiffness is considered as a good alternative. For that purpose, a non-linear static analysis of the structure with a load combination of $1.0D + 1.0L$ is performed in SAP2000 and the reduced stiffness of the structure after the analysis is used to conduct an eigenvalue analysis to extract the mode shapes and frequencies of this structure. Then the first 20-modes are used to perform a modal time history analysis for wind load. The result shows that the responses from modal analysis with “20-mode (reduced stiffness)” are comparable with that from the P- Δ analyses of Newmark-method.

Keywords – Tall Building, Time-history analysis, Wind tunnel

I. Introduction

As a background to potential numerical problems, consider the portal frame of Fig 1a, for which the 1st mode and 2nd mode deformed shapes are shown in Fig 1b and Fig 1c, respectively. Fig 1b shows that for the 1st mode shape, a conventional beam finite-element with a cubic displacement shape function is used to model the members of this portal frame gives a buckling load ($P_{cr} = 0.76\pi^2EI/L^2$) much larger than the exact value ($P_{cr} = 0.694\pi^2EI/L^2$); the differences are much greater and alarming for the second mode (2.55 vs. $4.56\pi^2EI/L^2$).

The reason for this difference is that in the stiffness matrix of the finite-element, an exact expression of the stiffness term $\frac{4EI}{L}\phi_3$, where $\phi_3 = \frac{1}{4} \frac{KL(\sin KL - KL \cos KL)}{2 - 2\cos KL - KL \sin KL}$, is reduced to $\frac{4EI}{L} \left(1 - \frac{P}{EI} \frac{2L^2}{15.4}\right)$ because of the cubic displacement shape assumed by the finite element formulation. Fig 2 shows a graphical comparison of the two expressions for ϕ_3 . This simple example shows that care needs to be exercised when using a finite-element approach to predict behaviour of structures that have significant contributions from higher-modes, such as tall flexible buildings. In general, for such large structures, higher modes calculated by FEs are not correct and many computed higher modes are not even physically meaningful. Thus, it is desirable (necessary) to filter out those incorrect or spurious higher modes.

For this work it is assumed that in the very near future the design of a structure for wind will be based on time history analyses using pressure history data from wind tunnel tests. For this approach, the choice of a time-step and the type of integration method for time history analysis is crucial. Typically, in seismic analysis, a time-step of 0.01 second is used when the duration of the strong ground motion is below one to two minutes. However, the choice of a very small time-step for wind analysis might not be wise as the duration of a wind

time history is typically in the order of tens of minutes to hours. This results in long computational times combined with larger and higher physical memory requirements if the model has a large number of sophisticated elements. This study explores the type of integration method suitable for wind analysis, with the aim of determining the best combination of accuracy and computational efficiency for the diagrid structure under study.

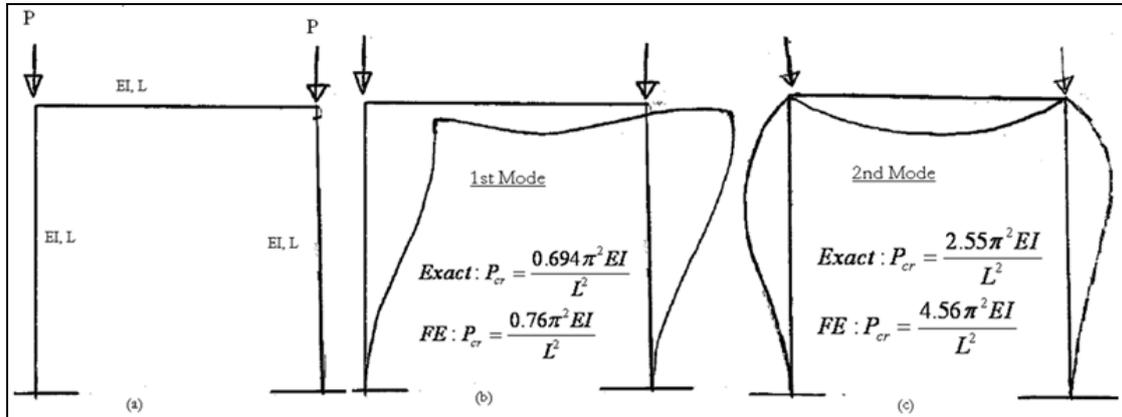


Figure 1: (a) Portal frame; (b) 1st Mode deformed shape; (c) 2nd Mode deformed shape

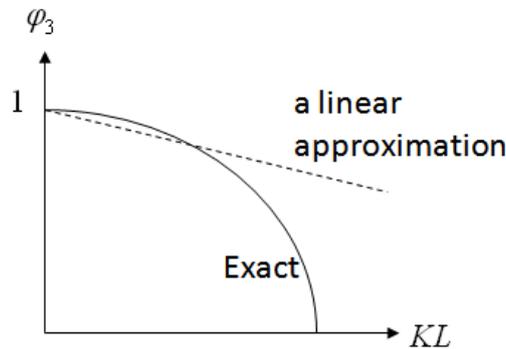


Figure 2: A linear approximation to the exact expression of ϕ_3

Error is inherent in any numerical solution of the equation of motions. Some numerical methods may predict that the displacement amplitude decays with time, although the system is undamped (which is termed as numerical/algorithmic damping), and that the natural period elongates or shortens. For a large structure, the use of an unconditionally stable algorithms is generally preferred over a conditionally stable one as long as the computation effort remains reasonable. For unconditionally stable algorithms, the step size may be selected independently of stability considerations and thus can result in a substantial saving of computational effort. In addition to being unconditionally stable, when only low-mode response is of interest, it is often advantageous for an algorithm to possess some form of numerical dissipation to damp out any spurious effects due to the high-frequency modes. Commonly used algorithms are the Newmark family of methods, the Wilson θ -method, and the Hilber-Hughes-Taylor (HHT) method, also commonly known as the α -method. The advantages and disadvantages of different algorithms are discussed with reference to several figures presented next (Hilber, 1976 [1]; Chung et al, 1993 [2]; Bathe, et al, 1973 [3]; Hoff, et al, 1989 [4]). Fig 3 shows plot of algorithmic damping ratios versus $\Delta t/T$ for α -method and some Newmark methods. It shows that the trapezoidal rule (Newmark method for which $\beta=0.25$ and $\gamma=0.5$) does not possess any algorithmic damping, which means this method will be unable to damp out spurious higher modes. The figure also shows that although a dissipative Newmark method ($\gamma>0.5$) possess algorithmic damping, it also results in higher algorithmic damping for low-frequency modes, thus strongly affecting the response of the structure. As can be seen from Fig 3, the α -method or HHT-method does not affect the low-frequency modes and also possesses algorithmic damping for the high-frequency modes. Fig 4 shows that the Wilson θ -method and the Houbolt method strongly affect low-frequency modes and thus may not be the suitable one for most of the structures.

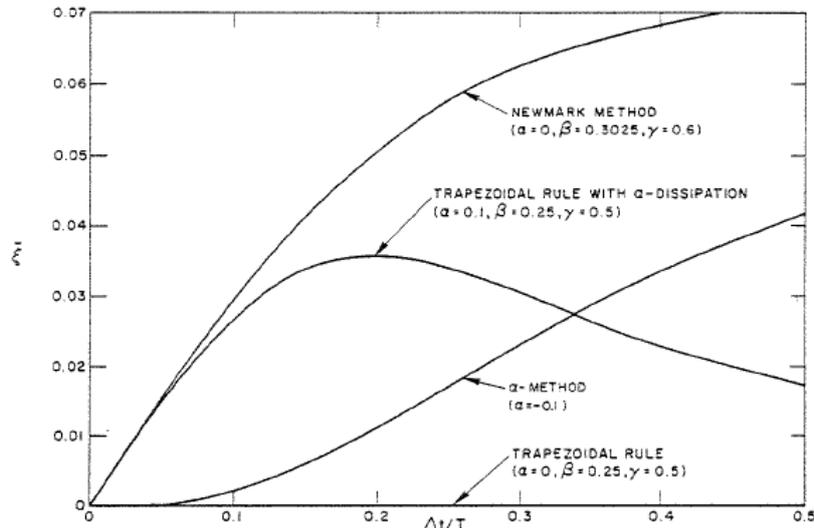


Figure 3: Algorithmic damping ratios versus $\Delta t/T$ for α -method and some Newmark schemes (adapted from Hilber, 1976 [1])

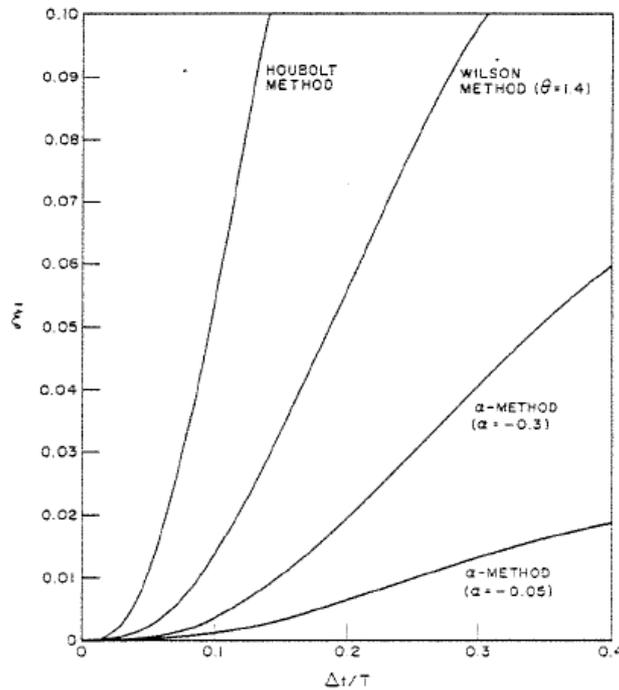


Figure 4: Algorithmic damping ratios versus $\Delta t/T$ for α -method and Houbolt and Wilson schemes (adapted from Hilber, 1976 [1])

Fig 5 shows the effect of viscous damping for the α -method. It indicates that the effect of viscous damping disappears for large $\Delta t/T$ thus has practically no dissipative effect on modes for which the value of $\Delta t/T$ is large. The same is true for the trapezoidal rule. Finally, in Fig 6 the relative period error is plotted versus $\Delta t/T$ for various algorithms.

In this paper, responses from the following types of analyses will be compared to choose a best suited numerical integration method for wind time history analysis of tall building:

- Modal-time history analysis (THA) considering the first 20, 43 and 192 modes of the building using Piecewise-Exact Integration methods,
- THA using the Hilber-Hughes-Taylor integration method, an approach used to verify direct-integration THA does not give noticeable error,
- Direct-integration THA using Hilber-Hughes-Taylor and Newmark integration methods without considering P- Δ effect,

- Direct-integration THA using Hilber-Hughes-Taylor and Newmark integration method considering P- Δ effect, and
- A simplified method that uses mode shapes of the structure derived from a reduced stiffness matrix found after running a static analysis with 1.0 D + 1.0 L. These mode shapes are then used to perform a Modal-THA considering the first 20 Modes of the building using Piecewise-Exact Integration method.

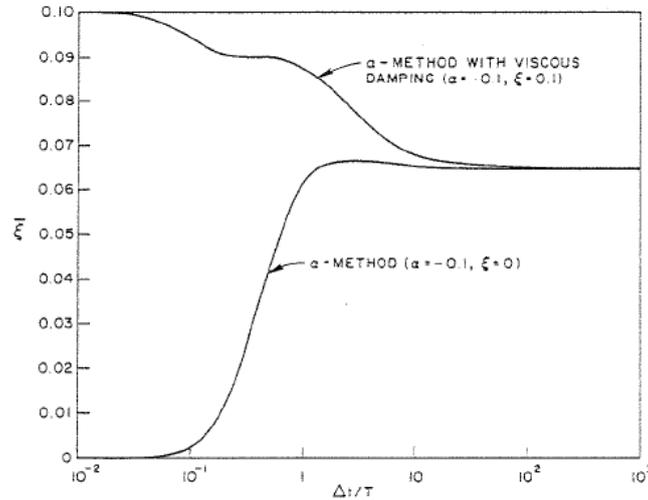


Figure 5: α -method with and without viscous damping (adapted from Hilber, 1976 [1])

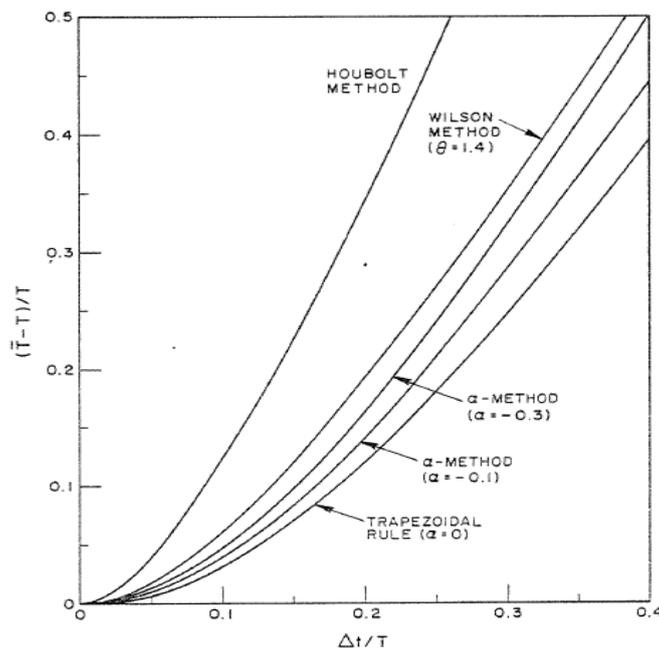


Figure 6: Relative period error versus $\Delta t/T$ for different algorithms (adapted from Hilber, 1976 [1])

II. Description of The Building and wind Tunnel Test

The initial wind data were obtained from the National Institute of Standards and Technology (NIST) from tests carried out by the Inter-University Research Centre on Building Aerodynamics and Wind Engineering, Boundary Layer Wind Tunnel, Prato, Italy (Venanzi, 2005[5]). The test was done on a 9cm:6cm:36.6cm model (i.e. 6:4:24.4 ratio) with total 120 pressure taps. A 3D view of the 64-story diagrid structure used in this study and which has similar dimensions is shown in Fig 7(a). The locations of the pressure taps around building perimeter are shown in Fig 7(b). Pressures at each of the 120 tap location was recorded simultaneously. The wind is acting perpendicular to the long face of the building as shown in Fig 8, where Cartesian axes are also defined along with the dimensions of the 64-story diagrid prototype building (Bhuiyan, 2011[6]).

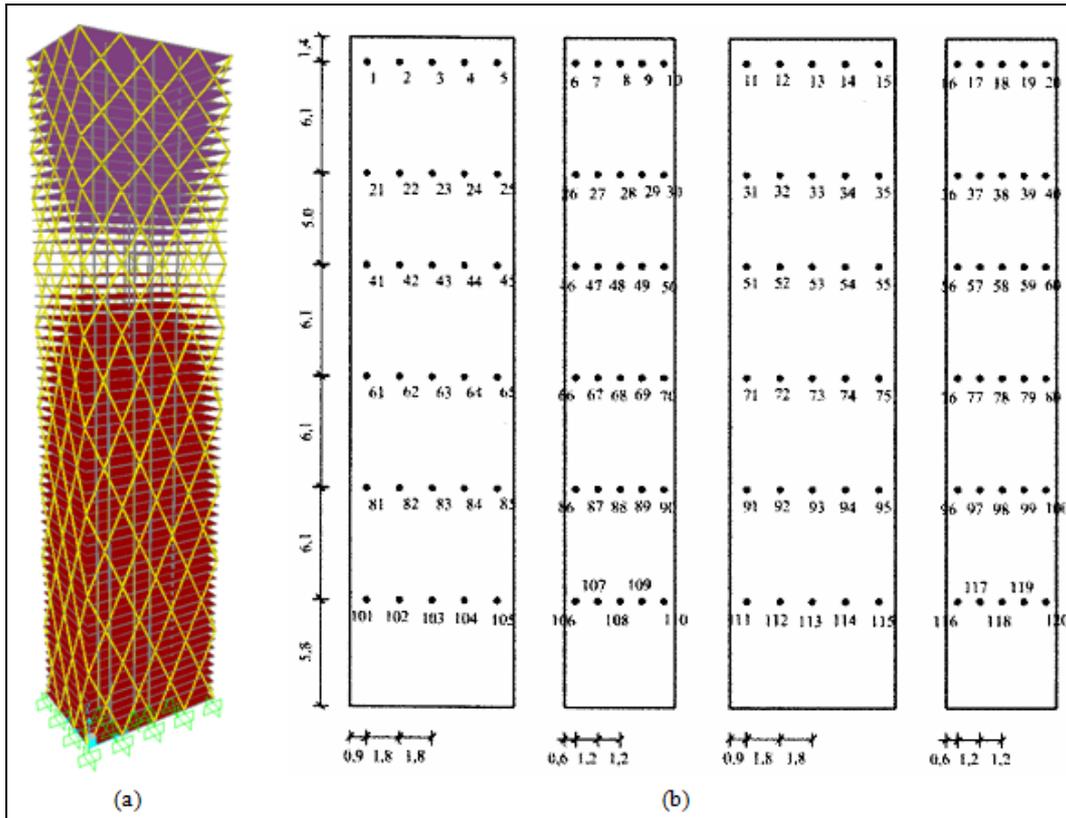


Figure 7: (a) 3-D view of 64-story Diagrid structure; (b) Tap location around the building faces for BLWT (adapted from NIST & CRIACIV, Prato, Italy)

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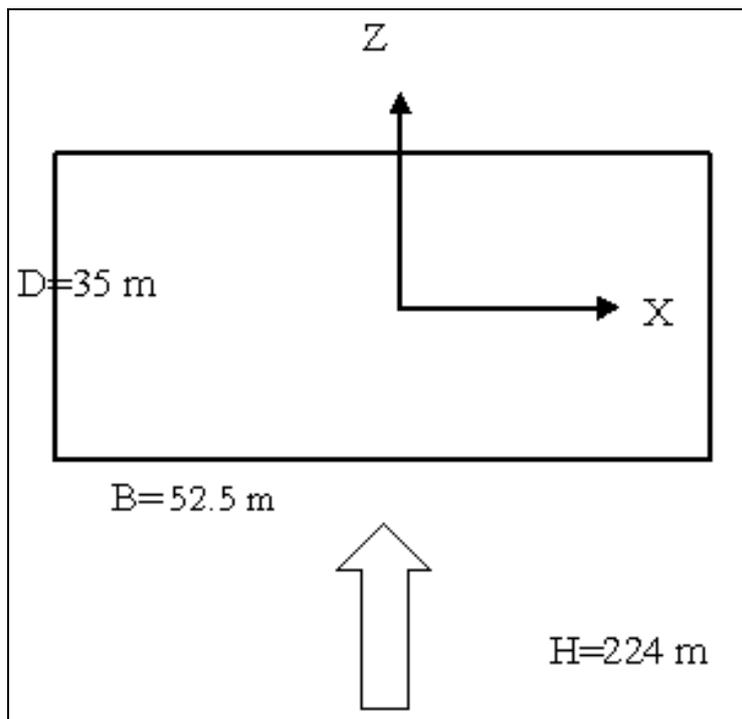


Figure 8: Building Dimensions and Wind Direction

III. Investigation and Results

From a wide variety of integration method, it was decided to investigate three methods that seem appropriate for 3D-time history analysis of tall buildings for wind. These are modal time history analysis, the Hilber-Hughes-Taylor (HHT) method or α -method with $\alpha=0.1$, and the Newmark method with $\beta=0.25$ and $\gamma=0.5$ (i.e., Trapezoidal Rule).

SAP2000 and the 64-story structure are used to conduct all the analyses of this section. As described in previous section, a BLWT pressure time history at 120 locations around the building envelope is used. As the purpose of this of the study is related to the investigation on numerical integration methods, the duration of the time history is arbitrarily limited to thirty-minutes.

The modal time history analysis provides an exact solution of the equation of motion. It is possible to limit number of modes to be included in the analysis, and to discard spurious higher modes which are not real or accurate. One major disadvantage of this method is that the P-delta effects cannot be captured directly. Modal time history analyses are performed considering 20-, 43- and 192-modes of the structure of this study. As the bulk of the energy of wind is in low frequency modes, it is expected that response of the structure considering 20-modes will be sufficiently accurate. The modal mass participation ratio for the first 20 modes are 94%, 96% and 95% in the X, Z and torsional directions, respectively. A modal mass participation ratio very close to 100% in X-, Z- and torsional directions is obtained when 192 modes are considered.

The Hilber-Hughes-Taylor (HHT) method is widely used by most of the commercial finite-element programs. Two 3D-THA without P-effects are performed using integration time-steps of 0.05 sec and 0.287 sec, respectively. For comparison, another 3D-THA is conducted including P-delta effects with a time-step of 0.05 sec.

While the Newmark method is a very good integration method, it does not have any damping-out for higher modes. Two 3D-THA are performed using 0.05 sec and 0.287 sec time-steps without P-delta effects for comparison with the HHT method. Similarly, a 3D-THA is conducted including P-delta effects with a time-step of 0.05 sec.

The results of these analyses are summarized in Fig 9 through 11, which compare story shear FX, story moment MX and story moment MZ, respectively. All the figures show that the response from modal analyses considering 20-, 43- and 192-modes overlap with each other. This type of outcome is expected because of the concentration of energy of wind in low-frequency modes. Response quantities from the HHT-method (for $\Delta t = 0.05$ sec) are also plotted in these figures. As the HHT-method possesses algorithmic damping, response values are lower when compared to those from modal analyses. Responses from Newmark-method, which match those of the modal analyses, are not plotted for clarity of the figures.

Table 1 presents the maximum displacement and acceleration at the roof level of the building for different integration methods. Roof displacements and acceleration for the modal analyses incorporating different number of modes are very similar. Observation of the table for the HHT- and Newmark-method with different time-steps indicate that there are some differences in response and one should use a small time-step if the HHT or Newmark-method is utilized. Analyses performed with both the HHT and Newmark-method considering P-delta effects show that the P-delta effect has considerable impact on the responses (for both displacement and acceleration). This result shows that it is necessary to account P-delta effects for wind analysis of tall buildings.

As the direct integration time history analysis required very large computation times and very large computer physical memory for a wind duration of hours, a modal analysis with reduced stiffness was considered as a good alternative. For that purpose, a non-linear static analysis of the structure with a load combination of 1.0D + 1.0L is performed in SAP2000 and the reduced stiffness of the structure after the analysis is used to conduct an eigenvalue analysis to extract the mode shapes and frequencies of this structure. Then the first 20-modes are used to perform a modal time history analysis for wind load. In Table 1, the row labeled "20-modes (reduced stiffness)" represents the results from this analysis. Observation of the table reveal that the responses from modal analysis with "20-mode (reduced stiffness)" are comparable with that from the P-delta analyses of Newmark-method. It is authors' recommendation to use modal time history analysis with reduced stiffness of the structure for wind analysis.

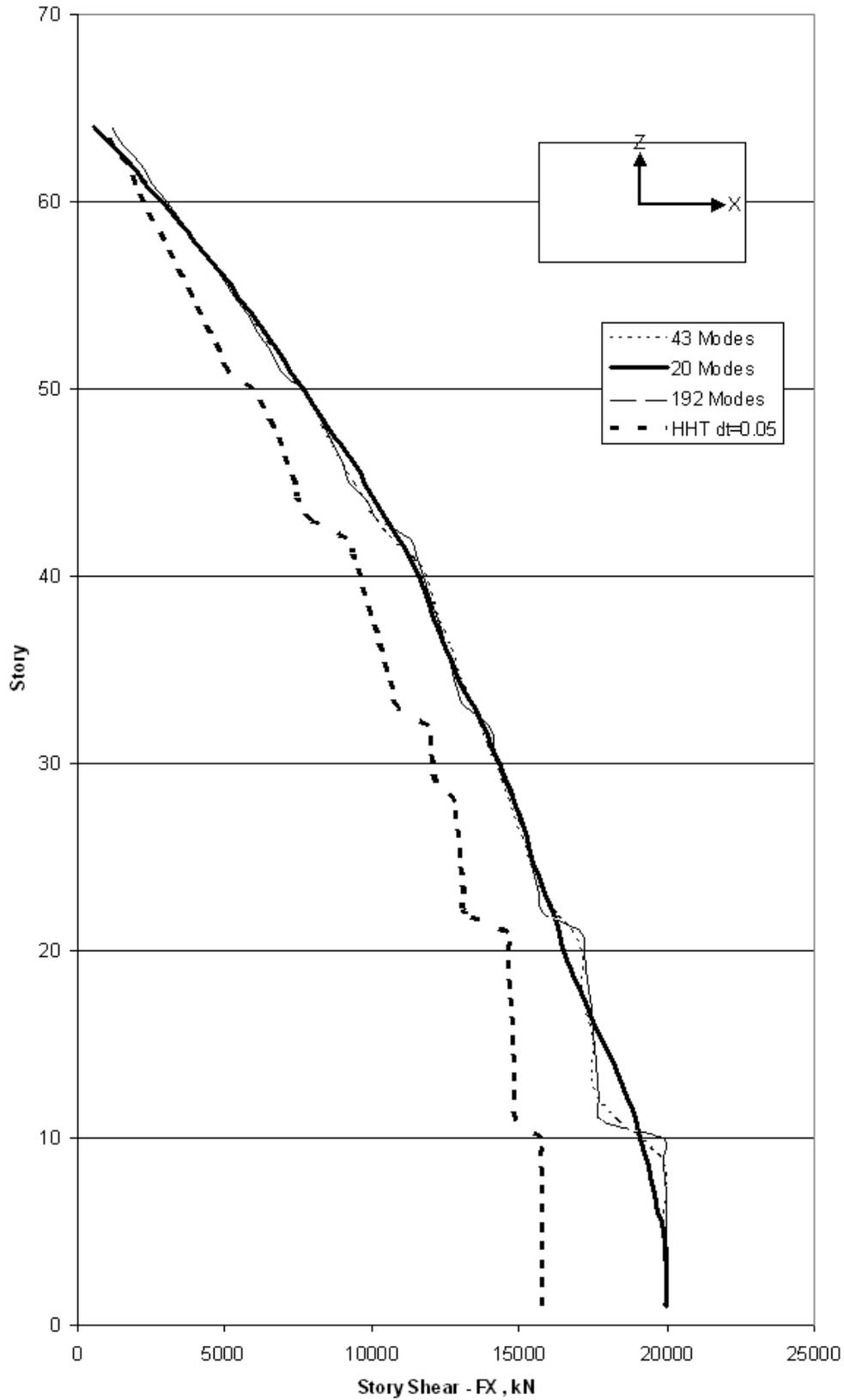


Figure 9: Comparison of different integration method – Story Shear FX

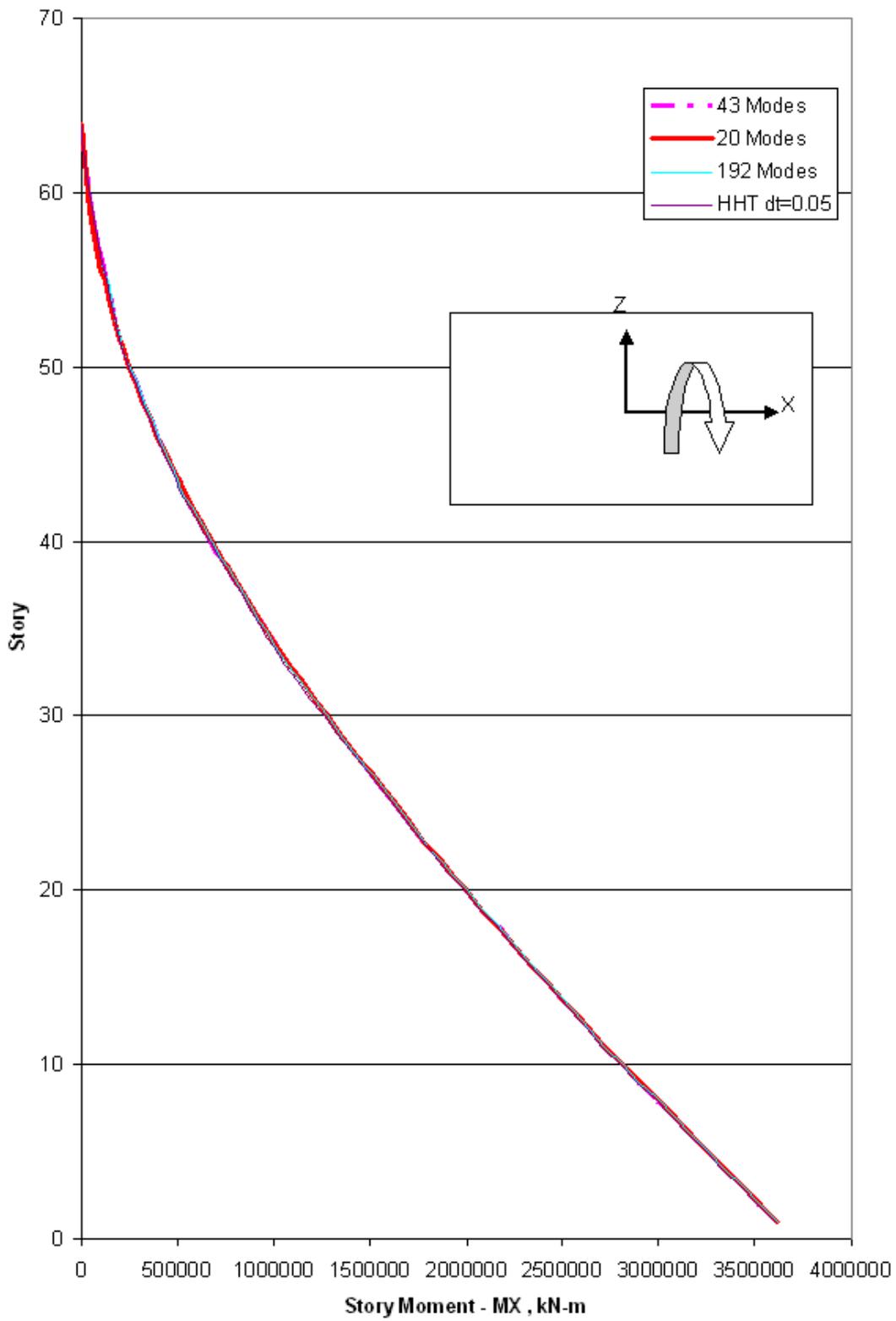


Figure 10: Comparison of different integration method – Story Moment M_X

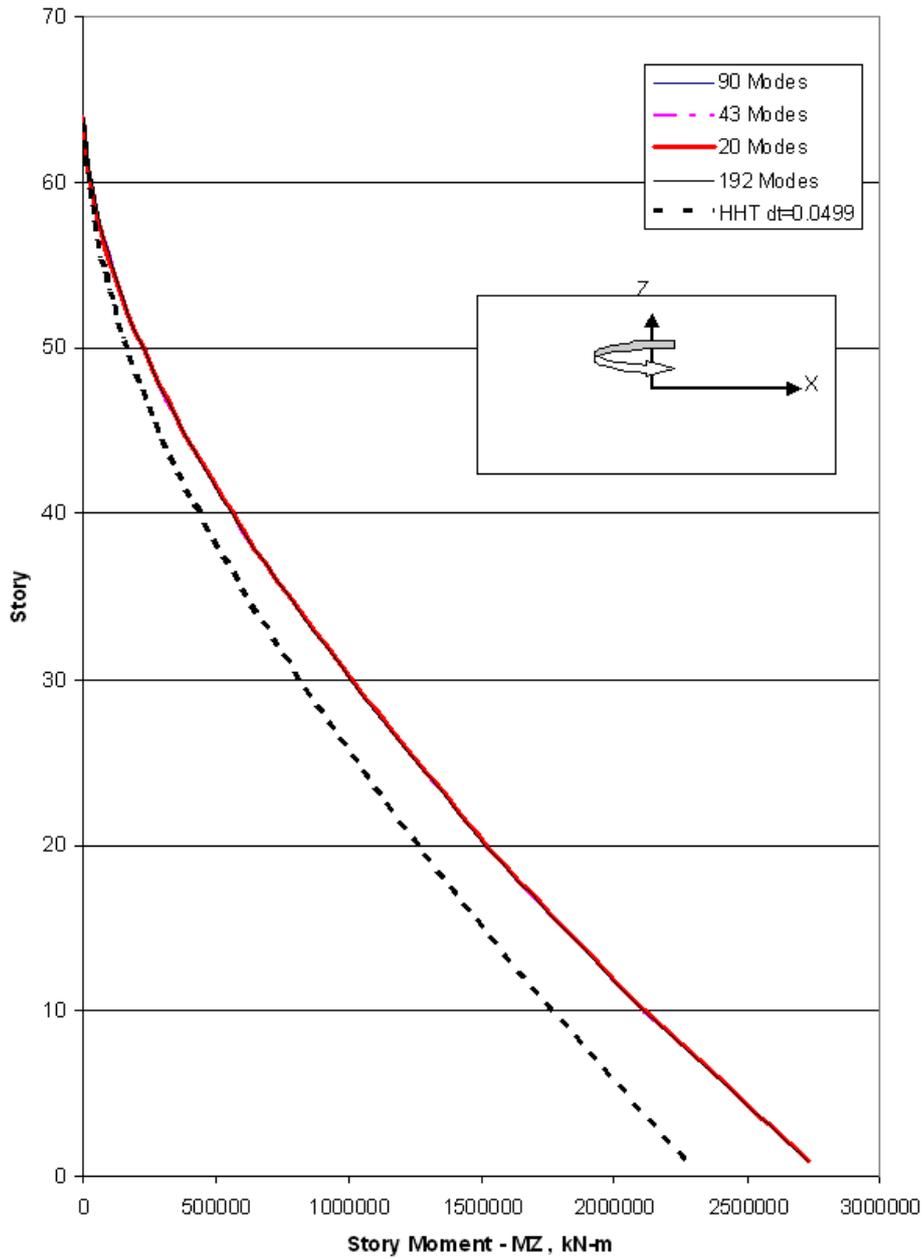


Figure 11: Comparison of different integration method – Story Moment MZ

Table 1: Maximum displacement and acceleration at roof level for different integration method

		UX (m)	UZ (m)	Acceleration-X (mili-g)	Acceleration-Z (mili-g)
Modal Analysis	20-modes	0.22	0.527	57.5	30.6
	43-modes	0.22	0.528	57.7	30.7
	192-modes	0.22	0.528	57.2	30.7
	20-modes (reduced stiffness)	0.267	0.564	62	31.4
HHT-method	dt = 0.05sec (w/o P-delta)	0.18	0.524	39.3	23.3
	dt=0.287sec (w/o P-delta)	0.14	0.488	23.8	17.3
	dt = 0.05sec (w/ P-delta)	0.21	0.550	44	24.3
Newmark method	dt = 0.05sec (w/o P-delta)	0.22	0.528	54.6	30.1
	dt=0.287sec (w/o P-delta)	0.257	0.545	59.2	27.9
	dt = 0.05sec (w/ P-delta)	0.267	0.565	60.9	31.3

IV. Conclusion

An investigation on the choice of numerical integration method for wind time history analysis was carried out to assess the most efficient and accurate algorithms for practical use. The results showed that first-order modal time history analysis with the first 20 modes captured the response properly for most serviceability conditions. P- Δ analyses with the HHT and Newmark methods showed that second-order effects are non-negligible, and a simplified procedure to account for this is proposed.

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