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# Some new exact Solutions for the nonlinear schrödinger equation

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**ABSTRACT :** *In this paper, we apply the generalized Bernoulli sub-ODE method to seek exact solutions for nonlinear evolution equations arsing in the theory of mathematical physics. For testing the validity of this method, we use it to derive exact solutions for the nonlinear Schrödinger (NLS) equation. As a result, some exact solutions for it are successfully found with the aid of the mathematical software Maple.*

**KEYWORDS :** *sub-ODE method, travelling wave solutions, exact solution, nonlinear evolution equation, nonlinear Schrödinger equation*

## I. INTRODUCTION

Research on solutions of NLEEs is a hot topic in the literature. So, the powerful and efficient methods to find analytical solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

Recently, seeking the exact solutions of nonlinear equations has getting more and more popular. Many approaches have been presented so far such as the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the nonlinear transform method [9], the inverse scattering transform [10], the Backlund transform [11,12], the Hirotas bilinear method [13,14], the generalized Riccati equation [15,16], the Jacobi elliptic function expansion [17,18], the complex hyperbolic function method [19-21], the generalized Bernoulli sub-ODE method [22,23] and so on.

In this paper, we present an application for the generalized Bernoulli sub-ODE method, and seek exact solutions for those nonlinear evolution equations arsing in the theory of mathematical physics.
The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent section, we will apply the method to find exact travelling wave solutions for the nonlinear Schrödinger equation. In the last section, some conclusions are presented.

## II. DESCRIPTION OF THE BERNOULLI SUB-ODE METHOD

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2 \tag{1}$$

where $\lambda \neq 0, G = G(\xi)$.

When $\mu \neq 0$, Eq. (1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + d e^{\lambda \xi}} \tag{2}$$

where $d$ is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables $x, y$ and $t$, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} \ldots\ldots) = 0 \tag{3}$$

where $u = u(x, y, t)$ is an unknown function, $P$ is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1.We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{4}$$

The travelling wave variable (4) permits us reducing Eq. (3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', \ldots\ldots) = 0 \tag{5}$$

Step 2. Suppose that the solution of (5) can be expressed by a polynomial in $G$ as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + \ldots\ldots \tag{6}$$

where $G = G(\xi)$ satisfies Eq. (1), and $\alpha_m, \alpha_{m-1}\ldots$ are constants to be determined later, $\alpha_m \neq 0$ . The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in (5).

Step 3. Substituting (6) into (5) and using (1), collecting all terms with the same order of $G$ together, the left-hand side of Eq. (5) is converted into another polynomial in $G$ . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, \ldots \lambda, \mu$ .

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (1), we can construct the travelling wave solutions of the nonlinear evolution equation (5).
In the subsequent sections we will illustrate the validity of the proposed method by applying it to solve several nonlinear evolution equations.
In the subsequent sections we will illustrate the proposed method in detail by applying it to Fitzhugh-Nagumo equation.

## III. APPLICATION OF THE BERNOULLI SUB-ODE METHOD FOR NLS EQUATION

In this section, we will consider the following NLS equation:

$$i\phi_t - \phi_{xx} + 2(|\phi|^2 - \rho^2)\phi = 0 \tag{7}$$

where $\phi$ is complex wave function and $\rho$ is a constant.

Since $\phi = \phi(x,t)$ in Eq. (7) is a complex function, we suppose that

$$\phi = u(\xi)\exp[i(\alpha x + \beta t)], \xi = k(x + 2\alpha t) \tag{8}$$

where the constants $\alpha, \beta, k$ can be determined later.

By using (8), (7) is converted into an ODE

$$(-\beta + \alpha^2 - 2\rho^2)u + 2u^3 - k^2 u'' = 0 \tag{9}$$

Suppose that the solution of (10) can be expressed by a polynomial in $G$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{10}$$

where $a_i$ are constants, and $G = G(\xi)$ satisfies Eq. (2.1).

Balancing the order of $u^3$ and $u''$ in Eq. (9), we obtain that $3m = m + 2 \Rightarrow m = 1$ .So Eq. (10) can be rewritten as

$$u(\xi) = a_1 G + a_0, a_1 \neq 0 \tag{11}$$

$a_1, a_0$ are constants to be determined later.

Substituting (11) into (9) and collecting all the terms

with the same power of $G$ together, the left-hand side of Eq.(9) is converted into another polynomial in $G$ .
Equating each coefficient to zero, yields a set of simul-taneous algebraic equations as follows:

$$G^0 : -\beta a_0 + \alpha^2 a_0 - 2\rho^2 a_0 + 2a_0^3 = 0$$

$$G^1 : -\beta a_1 - 2\rho^2 a_1 + \alpha^2 a_1 - k^2 a_1 \lambda^2 + 6a_0^2 a_1 = 0$$

$$G^2 : 6a_1^2 a_0 + 3k^2 \lambda a_1 \mu = 0$$

$$G^3 : 2a_1^3 - 2k^2 a_1 \mu^2 = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_1 = -k\mu, a_0 = \frac{1}{2}k\lambda, \beta = \alpha^2 - 2\rho^2 + 2a_0^2 \tag{12}$$

Substituting (14) into (13), we have

$$u_1(\xi) = -k\mu G + \frac{1}{2}k\lambda,\ \xi = k(x+2\alpha t). \tag{13}$$

Combining with Eq. (2.2) and considering $u = v^{-\frac{1}{n-1}}$, we can obtain the travelling wave solutions of (7) as follows:

$$\phi_1(\xi) = [-k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}) + \frac{1}{2}k\lambda]\exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2a_0^2)t)] \tag{14}$$

where $d$ is an arbitrary constant. Then we have

$$\phi_1(x,t) = [-k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k(x+2\alpha t)}}) + \frac{1}{2}k\lambda]\exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2a_0^2)t)] \tag{15}$$

Case 2:

$$a_1 = k\mu,\ a_0 = -\frac{1}{2}k\lambda,\ \beta = \alpha^2 - 2\rho^2 + 2a_0^2 \tag{16}$$

Substituting (14) into (13), we have

$$u_2(\xi) = k\mu G - \frac{1}{2}k\lambda,\ \xi = k(x+2\alpha t). \tag{17}$$

Combining with Eq. (2.2) and considering $u = v^{-\frac{1}{n-1}}$, we can obtain the travelling wave solutions of (7) as follows:

$$\phi_2(\xi) = [k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}) - \frac{1}{2}k\lambda]\exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2a_0^2)t)] \tag{18}$$

where $d$ is an arbitrary constant.
Then we have

$$\phi_2(x,t) = [-k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k(x+2\alpha t)}}) + \frac{1}{2}k\lambda]\exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2a_0^2)t)] \tag{19}$$

## IV. CONCLUSION

In the present work, we have found some new travelling wave solutions for the nonlinear Schrödinger (NLS) equation by the Bernoulli sub-ODE method. As one can see, this method is straight ward, and can be fulfilled with the aid of the mathematical software Maple. Being concise and simple, this method is one of the most effective approaches handling nonlinear evolution equations, and can be used to many other nonlinear problems.

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