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Bifurcation and Stability Analysis in Dynamics of Prey-Predator Model with Holling Type IV Functional Response and Intra-Specific Competition

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ABSTRACT: *This paper deals with the dynamical behaviour of discrete Prey-Predator model with Holling type IV involving intra-specific competition. This model represents mathematically by nonlinear differential equations. The existence, uniqueness and boundedness solutions of this model were investigated. The locally asymptotic stability conditions of all possible equilibrium points were obtained. The stability/instability of nonnegative equilibria and associated bifurcation are investigated by analyzing the characteristic equations. Moreover, bifurcation diagram are obtained for different values of parameters of this model. Finally, numerical simulation was used to study the global and rich dynamics of this model.*

KEYWORDS: *Differential equations, Prey-predator model, Functional response, Holling type IV functional response, Stability analysis, Bifurcation.*

I. INTRODUCTION

Discrete time models give rise to more efficient computational models for numerical simulations and it exhibits more plentiful dynamical behaviours than a continuous time model of the same type. There has been growing interest in the study of prey-predator discrete time models described by differential equations. In ecology, predator-prey or plant herbivore models can be formulated as discrete time models. It is well known that one of the dominant themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey. One of the important factors which affect the dynamical properties of biological and mathematical models is the functional response. The formulation of a predator-prey model critically depends on the form of the functional response that describes the amount of prey consumed per predator per unit of time, as well as the growth function of prey [1,16]. That is a functional response of the predator to the prey density in population dynamics refers to the change in the density of prey attached per unit time per predator as the prey density changes.

Two species models like Holling type II, III and IV of predator to its prey have been extensively discussed in the literature [2-7,10,17]. Leslie-Gower predator- prey model with variable delays, bifurcation analysis with time delay, global stability in a delayed diffusive system has been studied [9,13,15]. Limit cycles for a generalized Gause type predator- prey model with functional response, three tropic level food chain system with Holling type IV functional responses , the discrete Nicholson Bailey model with Holling type II functional response and global dynamical behavior of prey-predator system has been revisited [8,11,12,14].

The purpose of this paper is to study the dynamics of prey-predator model with Holling type IV function involving intra-specific competition. We prove that the model has bifurcation that is associated with intrinsic growth rate. The stability analysis that we carried out analytically has also been proved. The period-doubling or bifurcations exhibited by the discrete models can be attributed to the fact that ecological communities show several unstable dynamical states, which can change with very small perturbation. This paper is organized as follows: In section 2 we introduce the model. In section 3, we obtain the equilibrium points and the local stability conditions of the trivial and axial equilibrium points were investigated by using Lemma (2), when the prey population in system (3) is subject to a Holling type IV functional response. In section 4 we analysed the local and dynamical behaviour of the interior equilibrium point, when the prey population in system (3) is subject to a Holling type IV functional response. In section 5, we presented some numerical simulations, dynamical behaviour of the system and bifurcation diagrams supporting the theoretical stability results. Finally, the last section 6, is devoted to the conclusion and remarks. Diagrams are presented in Appendix.

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In this paper we consider the following classical Prey- Predator system:

$$\begin{cases} \dfrac{dx}{dt} = xq(x) - \alpha yp(x) \\ \dfrac{dy}{dt} = yp(x) - \beta y \end{cases} \quad (1)$$

$$x(0), y(0) > 0,$$

Where x, y represent the prey and predator density, respectively. $p(x)$ and $q(x)$ are so-called predator and prey functional response respectively. $\alpha,\ \beta > 0$ are the conversion and predator`s death rates, respectively. If $p(x) = \dfrac{mx}{a+x}$ refers to as Michaelis-Menten function or a Holling type – II function, where $m > 0$ denotes the maximal growth rate of the species, $a > 0$ is half-saturation constant. Another class of response function is Holling type-III $p(x) = \dfrac{mx^2}{a+x^2}$. In general the response function $p(x)$ satisfies the general hypothesis:

(A) $p(x)$ is continuously differentiable function defined on $[0, \infty)$ and satisfies $p(0) = 0,\ p^{|}(x) > 0$, and $\lim_{x\to\infty} p(x) = m < \infty.$ The inherent assumption in (A) is that $p(x)$ is monotonic, which is true in many predator-prey interactions. However, there is experimental and observational evidence that indicates that this need not always be the case, for example, in the cases of "inhibition" in microbial dynamics and "group defence" in population dynamics. To model such an inhibitory effect, Holling type-IV function $p(x) = \dfrac{mx}{a+x^2}$ found to be fit and it is simpler since it involves only two parameters. The Holling type – IV function otherwise known as Monod-Haldane function which is used in our model. The simplified Monod-Haldane or Holling type- IV functional is a modification of the Holling type-III function [11]. In this paper, we focus on prey-predator system with Holling type –IV by introducing intra-specific competition and establish results for boundedness, existence of a positively invariant and the locally asymptotical stability of coexisting interior equilibrium.

II. THE MODEL

The prey-predator systems have been discussed widely in many decades. In literature many studies considered the prey-predator with functional responses. However, considerable evidence that some prey or predator species have functional response because of the environmental factors. It is more appropriate to add the functional responses to these models in such circumstances. For instance, a system suggested in Eqn.(1), where $x(t)$ and $y(t)$ represent densities or biomasses of the prey species and predator species respectively; $p(x)$ and $q(x)$ are the intrinsic growth rates of the predator and prey respectively; $\alpha,\ \beta$ are the death rates of prey and predator respectively.

If $p(x) = \dfrac{mx}{1+x^2}$ and $q(x) = ax(1-x)$, in $p(x)$ assuming $a = 1$ in general function, that is where a is the half-saturation constant in the Holling type IV functional response, then Eq.(1) becomes

$$\begin{cases} \dfrac{dx}{dt} = x\left(a(1-x) - \dfrac{\alpha my}{1+x^2}\right) \\ \dfrac{dy}{dt} = y\left(\dfrac{mx}{1+x^2} - \beta\right) \end{cases} \quad (2)$$

Here a, α, β, m are all positive parameters.

Now introducing intra-specific competition, the Eq.(2) becomes

$$\frac{dx}{dt} = x\left(a - bx - \frac{\alpha m y}{1+x^2}\right)$$
$$\frac{dy}{dt} = y\left(\frac{e\alpha m x}{1+x^2} - \beta - \delta y\right) \quad (3)$$

with $x(0), y(0) > 0$ and $\alpha, \beta, \delta, m, a, b, e$ are all positive constants.

Where a is the intrinsic growth rate of the prey population; β is the intrinsic death rate of the predator population; b is strength of intra-specific competition among prey species; δ is strength of intra-specific competition among predator species; m is direct measure of predator immunity from the prey; α is maximum attack rate of prey by predator and finally e represents the conversion rate.

III. EXISTENCE AND LOCAL STABILITY ANALYSIS WITH PERSISTENCE

In this section, we first determine the existence of the fixed points of the differential equations (3), and then we investigate their stability by calculating the eigen values for the variation matrix of (3) at each fixed point. To determine the fixed points, the equilibrium are solutions of the pair of equations below:

$$x\left(a - bx - \frac{\alpha m y}{1+x^2}\right) = 0$$
$$y\left(\frac{e\alpha m x}{1+x^2} - \beta - \delta y\right) = 0 \quad (4)$$

By simple computation of the above algebraic system, it was found that there are three nonnegative fixed points:

(i) $E_0 = (0,0)$ is the trivial equilibrium point always exists.

(ii) $E_1 = \left(\frac{a}{b}, 0\right)$ is the axial fixed point always exists, as the prey population grows to the carrying capacity in the absence of predation.

(iii) $E_2 = \left(x^*, y^*\right)$ is the positive equilibrium point exists in the interior of the first quadrant if and only if there is a positive solution to the following algebraic nonlinear equations

We have the polynomial form of five and three degrees.

$$x^* = B_5 x^5 + B_4 x^4 + B_3 x^3 + B_2 x^2 + B_1 x + B_0$$
$$y^* = A_3 x^3 + A_2 x^2 + A_1 x + A_0 \quad (5)$$

Where

$$B_5 = \frac{-b}{e\alpha^2 m^2}, B_4 = \frac{a}{e\alpha^2 m^2}, B_3 = \frac{-2b}{e\alpha^2 m^2}, B_2 = \frac{2a}{e\alpha^2 m^2} + \frac{d}{e\alpha m}, B_1 = \frac{-b}{e\alpha^2 m^2}, B_0 = \frac{a}{e\alpha^2 m^2} + \frac{d}{e\alpha m}$$

and

$$A_3 = \frac{-b}{\alpha m}, A_2 = \frac{a}{\alpha m}, A_1 = \frac{-b}{\alpha m}, A_0 = \frac{a}{\alpha m}$$

Remark 1: There is no equilibrium point on $y-$axis as the predator population dies in the absence of its prey.

Lemma: For all parameters values, Eqn.(3) has fixed points, the boundary fixed point and the positive fixed point $\left(x^*, y^*\right)$, where x^*, y^* satisfy

$$\begin{cases} a - bx = \dfrac{\alpha m y}{x^2+1} \\ \dfrac{e\alpha m x y}{x^2+1} = \beta + \delta y \end{cases} \qquad (6)$$

Now we study the stability of these fixed points. Note that the local stability of a fixed point (x,y) is determined by the modules of Eigen values of the characteristic equation at the fixed point.

The Jacobian matrix J of the map (3) evaluated at any point (x,y) is given by

$$J(x,y) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{12} \end{pmatrix} \qquad (7)$$

Where $a_{11} = a - 2bx - \dfrac{\alpha m y\left(1-x^2\right)}{\left(1+x^2\right)^2}$; $a_{12} = -\dfrac{\alpha m x}{1+x^2}$

$a_{21} = \dfrac{e\alpha m y\left(1-x^2\right)}{\left(1+x^2\right)^2}$; $a_{22} = \dfrac{e\alpha m x}{1+x^2} - \beta - 2\delta y$

and the characteristic equation of the Jacobian matrix $J(x,y)$ can be written as

$\lambda^2 + p(x,y)\lambda + q(x,y) = 0$,

Where

$p(x,y) = -\left(a_{11} + a_{22}\right)$, $q(x,y) = a_{11}a_{22} - a_{12}a_{21}$.

In order to discuss the stability of the fixed points, we also need the following lemma, which can be easily proved by the relations between roots and coefficients of a quadratic equation.

Theorem: Let $F(\lambda) = \lambda^2 + P\lambda + Q$. Suppose that $F(1) > 0$, λ_1, λ_2 are two roots of $F(\lambda) = 0$. Then

(i) $|\lambda_1| < 1$ and $|\lambda_2| < 1$ if and only if $F(-1) > 0$ and $Q < 1$;

(ii) $|\lambda_1| < 1$ and $|\lambda_2| > 1$ (or $|\lambda_1| > 1$ and $|\lambda_2| < 1$) if and only if $F(-1) < 0$;

(iii) $|\lambda_1| > 1$ and $|\lambda_2| > 1$ if and only if $F(-1) > 0$ and $Q > 1$;

(iv) $\lambda_1 = -1$ and $|\lambda_2| \neq 1$ if and only if $F(-1) = 0$ and $P \neq 0, 2$;

(v) λ_1 and λ_2 are complex and $|\lambda_1| = 1$ and $|\lambda_2| = 1$ if and only if $P^2 - 4Q < 0$ and $Q = 1$.

Let λ_1 and λ_2 be two roots of ( ), which are called Eigen values of the fixed point (x,y). We recall some definitions of topological types for a fixed point (x,y). A fixed point (x,y) is called a sink if $|\lambda_1| < 1$ and $|\lambda_2| < 1$, so the sink is locally asymptotic stable. (x,y) is called a source if $|\lambda_1| > 1$ and $|\lambda_2| > 1$, so the source is locally un stable. (x,y) is called a saddle if $|\lambda_1| > 1$ and $|\lambda_2| < 1$ (or $|\lambda_1| < 1$ and $|\lambda_2| > 1$). And (x,y) is called non-hyperbolic if either $|\lambda_1| = 1$ and $|\lambda_2| = 1$.

Proposition 1: The Eigen values of the trivial fixed point $E_0 = (0,0)$ is locally asymptotically stable if $a, \beta < 1$ (i.e.,) E_0 is sink point, otherwise unstable if $a, \beta > 1$, and also E_0 is saddle point if $a > 1,\ \beta < 1$, E_0 is non-hyperbolic point if $a = 1,\ \beta = 1$.

Proof: In order to prove this result, we estimate the Eigen values of Jacobian matrix J at $E_0 = (0,0)$. On substituting (x, y) values in (7) we get the Jacobian matrix for E_0

$$J_0(0,0) = \begin{pmatrix} a & 0 \\ 0 & -\beta \end{pmatrix}$$

Hence the Eigen values of the matrix are λ_1=a, λ_2=-β

Thus it is clear that by Theorem, E_0 is sink point if $|\lambda_{1,2}|<1 \Rightarrow a, \beta < 1$, that is E_0 is locally asymptotically stable. E_0 is unstable (i.e.,) source if $|\lambda_{1,2}|>1 \Rightarrow a, \beta > 1$.

And also E_0 is saddle point if $|\lambda_1|>1,\ |\lambda_2|<1 \Rightarrow a>1,\ \beta<1, E_0$ is non-hyperbolic point if $|\lambda_1|=1$ or $|\lambda_2|=1 \Rightarrow a=1$ or $\beta=1$.

Proposition 2: The fixed point $E_1 = \left(\frac{a}{b}, 0\right)$ is locally asymptotically stable, that is sink if $a<1,\ \beta > \frac{em\alpha}{a^2+b^2}$; E_1 is locally unstable, that is source if $a>1,\ \beta < \frac{em\alpha}{a^2+b^2}$; E_1 is a saddle point if $a>1,\ \beta > \frac{em\alpha}{a^2+b^2}$ and E_1 is non-hyperbolic point if either $a=1$ or $\beta = \frac{em\alpha}{a^2+b^2} - 1$.

Proof: One can easily see that the Jacobian matrix at E_1 is

$$J_1\left(\frac{a}{b}, 0\right) = \begin{pmatrix} -a & \frac{-\alpha am}{a^2+b^2} \\ 0 & \frac{aem\alpha}{a^2+b^2} - \beta \end{pmatrix}$$

Hence the Eigen values of the matrix are

$$|\lambda_1| = a,\ |\lambda_2| = \frac{aem\alpha}{a^2+b^2} - \beta$$

By using Theorem, it is easy to see that, E_1 is a sink if $a<1$ and $\beta > \frac{em\alpha}{a^2+b^2}$; E_1 is a source if $a>1$ and $\beta < \frac{em\alpha}{a^2+b^2}$; E_1 is a saddle if $a>1$ and $\beta > \frac{em\alpha}{a^2+b^2}$; and E_1 is a saddle if either $a=1$ or $\beta = \frac{em\alpha}{a^2+b^2} - 1$.

Remark 2: If $\lambda^2 - Tr(J_2)\lambda + Det(J_2) = 0$, then the necessary and sufficient condition for linear stability are $Tr(J_2) < 0$ and $Det(J_2) > 0$.

4. Local Stability and Dynamic Behaviour around Interior Fixed Point E_2

Now we investigate the local stability and bifurcations of interior fixed point E_2. The Jacobian matrix at E_2 is of the form

$$J_2(x^*,y^*)=\begin{pmatrix} a-2bx^*-\dfrac{\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2} & -\dfrac{\alpha mx^*}{1+x^{*2}} \\ \dfrac{e\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2} & \dfrac{e\alpha mx}{1+x^{*2}}-\beta-2\delta y^* \end{pmatrix} \quad (8)$$

Its characteristic equation is $F(\lambda)=\lambda^2-Tr(J_2)\lambda+Det(J_2)=0$ where Tr is the trace and Det is the determinant of the Jacobian matrix $J(E_2)$ defines in Eq.(8), where

$$Tr(J_2) = a-2bx^*-\frac{\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2}+\frac{e\alpha mx}{1+x^{*2}}-\beta-2\delta y^* = \mathrm{G}_1+\mathrm{G}_2$$

and

$$Det(J_2) = \left[a-2bx^*-\frac{\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2}\right]\left[\frac{e\alpha mx}{1+x^{*2}}-\beta-2\delta y^*\right]+\frac{e\alpha^2m^2x^*y^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^3} = \mathrm{G}_1.\mathrm{G}_2+\mathrm{G}_3$$

$$\mathrm{G}_1=a-2bx^*-\frac{\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2}\ ,\quad \mathrm{G}_2=\frac{e\alpha mx}{1+x^{*2}}-\beta-2\delta y^*$$

and $$\mathrm{G}_3=\frac{e\alpha^2m^2x^*y^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^3}$$

By Remark 2, E_2 is stable if $\mathrm{G}_1+\mathrm{G}_2<0$ and $\mathrm{G}_1.\mathrm{G}_2+\mathrm{G}_3>0$ that is

E_2 is stable if

$$\frac{e\alpha mx}{1+x^{*2}}-\frac{\alpha my^*\left(1-x^{*2}\right)}{\left(1+x^{*2}\right)^2}-2\delta y^*-2bx^* < \beta-a \quad (9)$$

and

$$e < \frac{\left(\beta+2\delta y^*\right)\left[\alpha my^*\left(1-x^{*4}\right)-\left(a-2bx^*\right)\left(1+x^{*2}\right)^3\right]}{\alpha mx^*\left[2\alpha my^*\left(1-x^{*2}\right)-\left(1+x^{*2}\right)^2\left(a-2bx^*\right)\right]} \quad (10)$$

If both equations (9) and (10) are satisfied, then the interior equilibrium point will be stable.

IV. NUMERICAL SIMULATION

In this section, we undertake the numerical simulations of the prey-predator system (3) for the case when there is intra-specific competition with Holling type IV functional response. In the sequel, we plot diagrams for the prey system, the trivial and axial equilibrium points and also we present the bifurcation diagrams of the model (3) that have been obtained with data from 500 iterations with time-step of 0.005 units. The bifurcation diagrams are presented with the presence of predator and in the absence of predator. The plots have been generated using MATLAB 7. The prey-predator system with Holling type IV functional response and intra-specific competition exhibits a variety of dynamical behaviour in respect of the population size. The population shows several equilibrium states, and for certain higher values of the parameters there can be infinite number of such possibilities so far as the population size is concerned. This implies that for a particular species, if the intrinsic growth rate is high, in the presence of a high intra-specific competition, there is a possibility that it will show sudden spurts or drops (fig 2) in the population for seemingly minute changes in environmental conditions. The dynamics vary from steady (equilibrium) state to chaotic through a hierarchy of bifurcations. This chain of dynamics is exhibited in the bifurcation diagram of prey system x (Fig.6-10) for the different values of a (intrinsic growth rate).

Fig. 1. Shows a plot for prey equation when a=0.04; b=0.4; e=0.75; m=0.75; Fig.2 and Fig.3 Shows that sudden spurts or drops and a line diagram at the equilibrium point $E_0 = (0,0)$ is local asymptotically stable, if a=0.9, β =0.7, then the Eigen value becomes $|\lambda_1|$=0.9 and $|\lambda_2|$=0.7, which implies $|\lambda_{1,2}|<1$, that is E_0 is local asymptotically stable. Fig.4 Shows the diagram at the equilibrium point $E_1 = \left(\frac{a}{b}, 0\right)$ is local asymptotically stable, if a=0.9, b=0.2, α=0.5, m=0.75, e=0.75, δ =0.01, β =0.01, then the Eigen value becomes $|\lambda_1|$=0.9 and $|\lambda_2|$=0.0496, which implies $|\lambda_{1,2}|<1$, that is E_1 is local asymptotically stable. Fig.5 Shows the diagram at the equilibrium point $E_1 = \left(\frac{a}{b}, 0\right)$ is un stable, if a=4, b=0.9, α=0.5, m=0.75, e=0.75, δ =0.01, β =0.01, then the Eigen value becomes $|\lambda_1|$= 4 and $|\lambda_2|$= 0.0502, which implies $|\lambda_1|>1$, and $|\lambda_2|<1$, (i.e.,) E_1 is un stable.

Fig.6 (See appendix) Shows that there is no period-doubling bifurcation for intrinsic growth rate a=0 to 2, with only one predator and Fig.7 shows the bifurcation that bifurcates 2 cycles when the intrinsic growth rate=3 with one predator and the prey population bifurcates 4 cycles at 3.5. Fig.8 shows when intrinsic growth rate=0 to 4 with one predator, the prey population bifurcate 2 cycle at a=3 and bifurcate 4 cycles at a=3.5 and chaos after a= 3.5 that is increasing the parameters effectively makes the bounds on the system tighter and pushes it from stability towards unstable behaviour. This unstability manifests itself as a period-doubling bifurcation as a result of which the single equilibrium level of the population splits into two and the population starts oscillating between two levels which are quite different in their relative magnitudes. As we keep on increasing the parameters, these levels individually split up more and more frequently, until all order is lost and we found an infinite number of possible equilibrium states visited by the population. At this point, the population behaviour seems to lose any stablility. This appearance of nonperiodic behaviour from equilibrium population levels may be referred to as the "period-doubling route to chaos", the non periodic dynamics being described as *chaotic(*Fig.8*)*.

Fig.9 shows when the intrinsic growth rate 0 to 3.5 in the absence of predator bifurcates 2 cycles if $a > 3$. Fig.10 shows when intrinsic growth rate= 0 to 4, prey population bifurcates 2 cycles if $a > 3$ and bifurcates 4 cycles if $a > 3.5$ and if $a > 3.7$ prey population behaves like chaos.

V. CONCLUSION

In this paper, we investigated the complex behaviours of two species prey- predator system as a discrete-time dynamical system with Holling type IV functional response and intra-specific competition in the closed first quadrant, and showed that the unique positive fixed point of system (3) can undergo bifurcation and chaos. Bifurcation diagrams have shown that there exists much more interesting dynamical and complex behaviour for system (3) including periodic doubling cascade, periodic windows and chaos. All these results showed that for richer dynamical behaviour of the discrete model (3) under periodical perturbations compared to the continuous model. The system is examined via the techniques of local stability analysis of the equilibrium points from which we obtain the bifurcation criterion.

The numerical simulation of the population size shows a succession of period-doubling bifurcations leading up to chaos. The effect of intra-specific competition with Holling type IV functional response on the model depends on the value of the intrinsic growth rate. For values corresponding to the stable system dynamics, the population undergoes a linear change. However, for values of the intrinsic growth rate which makes the system dynamics bifurcates. It may thus be concluded that the stability properties of the system could switch with the Holling type IV functional response with intra-specific competition that is incorporated on different densities in the model.

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Appendix:

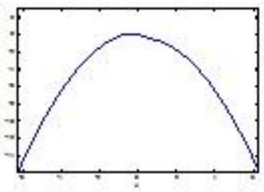

Fig.1
Prey system graph when $a=0.04, b=0.4; e=0.75; m=0.75$

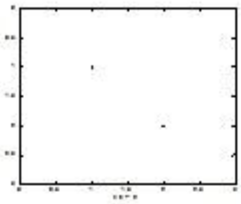

Fig.2
Spurts graph at ε_0 when $a=0.04, b=0.4; e=0.75; m=0.75$

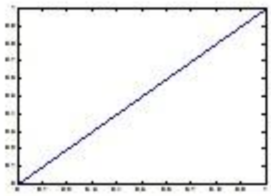

Fig.3
Line graph at ε_0 when $a=0.04, b=0.4; e=0.75; m=0.75$

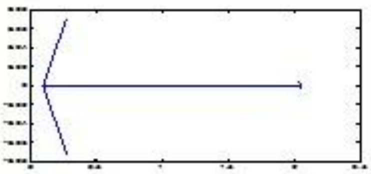

Fig.4
Stable graph, at ε_1 when $a=0.9$, $b=0.2$, $\alpha=0.5$, $m=0.75$, $e=0.75$, $\delta=0.01$, $\beta=0.01$, we get $|\lambda_1|=0.9$ and $|\lambda_2|=0.0496$, $\Rightarrow |\lambda_{1,2}|<1$

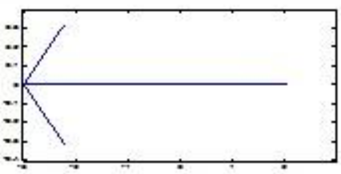

Fig.5
Unstable graph, at ε_1 when $a=4$, $b=0.9$, $\alpha=0.5$, $m=0.75$, $e=0.75$, $\delta=0.01$, $\beta=0.01$ we get $|\lambda_1|=4 \Rightarrow |\lambda_1|>1$, that is ε_1.

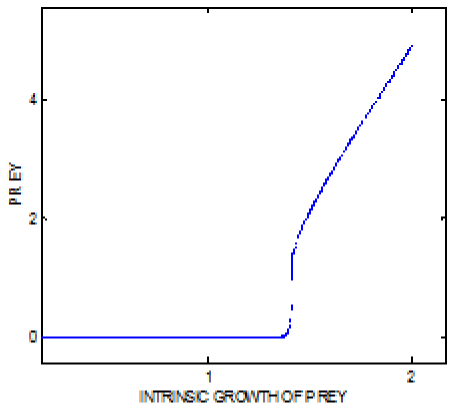

Fig.6
Bifurcation diagram for the prey system with intrinsic growth rate a, in the presence of predator, when a = 0 to 2

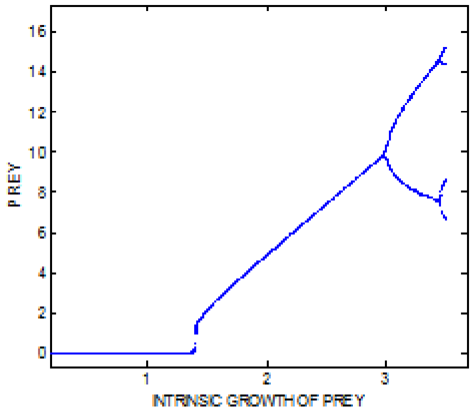

Fig.7
Bifurcation diagram for the prey system with intrinsic growth rate a, in the presence of predator, when a = 0 to 3.5

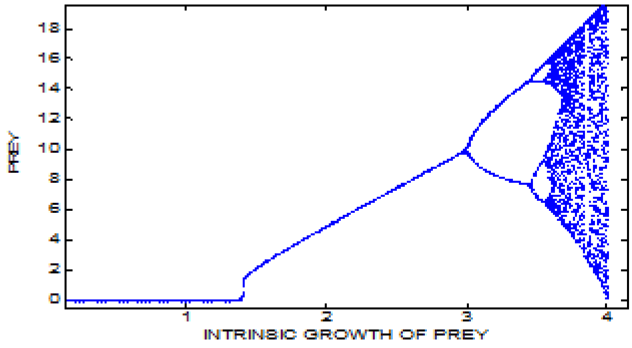

Fig.8
Bifurcation diagram for the prey system with intrinsic growth rate a, in the presence of predator when a = 0 to 4

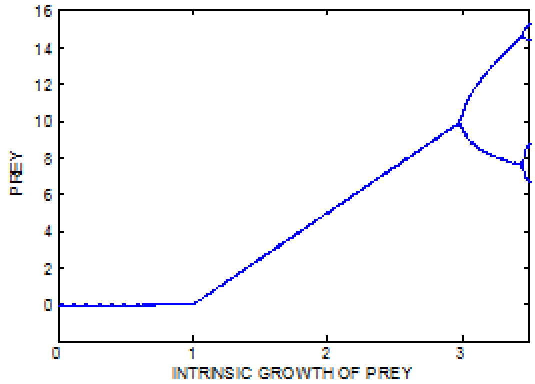

Fig.9 Bifurcation diagram for the prey system with intrinsic growth rate a, in the absence of predator when a = 0 to 3.5

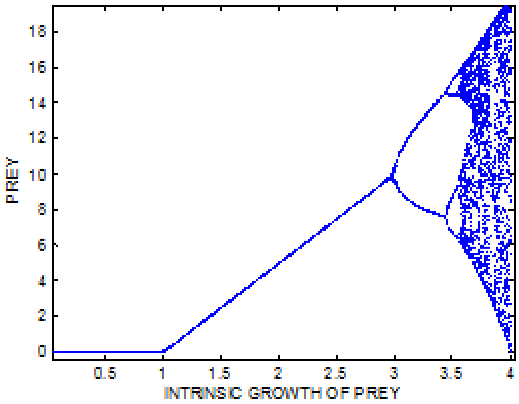

Fig.10 Bifurcation diagram for the prey system with intrinsic growth rate a, in the absence of predator when a = 0 to 4