

Restoring Corrupted Images Using Adaptive Fuzzy Filter

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ABSTRACT - The proposed histogram adaptive fuzzy (HAF) filter is particularly effective for removing highly impulsive noise while preserving edge sharpness. This is accomplished through a fuzzy smoothing filter constructed from a set of fuzzy IF-THEN rules, which alternate adaptively to minimize the output mean squared error as input histogram statistics change. An algorithm is developed to utilize input histogram to determine parameters for the fuzzy membership function. As compared to the conventional median filters (MF), the proposed method has the following merits: it is simple and it has superior performance compared to other existing ranked-order filters (including MF) for the full range of impulsive noise probability

Keywords - Histogram adaptive fuzzy (HAF) filter, median filters, membership functions.

I. INTRODUCTION

Median filtering (MF) is a nonlinear technique that is known for its effectiveness in removing impulsive noise while preserving edge sharpness. The 1-D MF is realized by passing a window over the input data and taking the median value of the data inside the window as the output associated with the center of the window.

In image processing applications, two-dimensional median filters have been used with some success [1]. The simplest way is to pass a 2-D window, such as a square mask, over the 2-D input image [2]. As with the 1-D MF, the pixels inside the window are ranked according to their gray intensity values, and the median value is taken as the output. Although noise suppression is obtainable by using MF, too much signal distortion is introduced, and features such as sharp corners as well as thin lines are lost. To overcome these problems, several variations of median filters have been developed specifically the detection –estimation based filter [3], which incorporated a statistical noise detection algorithm and the median filter for removal impulsive noise. Due to the lack of adaptability these median filters cannot perform well when $NP \leq 20\%$. Adaptive systems based on fuzzy or neural networks with data driven adjustable parameters have emerged as attractive alternatives [4]. In this category noise exclusive adaptive filter [5] have been developed..

Neural networks exploit their frameworks with many theorems and efficient training algorithms. They embed several input and output mappings on a black box web of connection weights. However, we cannot directly encode the simple rule of a spatial windowing operation, such as: “If most of the pixels in an input window are BRIGHT, then assign the output pixel intensity as BRIGHT.” On the other hand, fuzzy systems can directly encode structured knowledge. Fuzzy systems may invariably store banks of common-sense rules linguistically articulated by an expert or may adaptively infer and modify their fuzzy rules using representative symbols (e.g., DARK, BRIGHT) as well as numerical samples. Fuzzy systems and neural networks naturally combine and this combination produces an adaptive system. The hybrid neuro –fuzzy networks do not represent general means in restoring images [6]. used the adaptive fuzzy median filter (AFMF) with the backpropagation algorithm [7] to tune a set of randomly given initial membership functions.

In this paper, we propose a novel adaptive fuzzy filter (HAF) in which a set of memberships is estimated from the input histogram and used to achieve restoration without any training. In Section II the fuzzy inference rules related to the task of median filtering and the system architecture of HAF are introduced. In Section III a systematic algorithm based on conservation in the histogram potential to obtain a set of membership functions is implemented. In section V experiments are presented to characterize HAF as well as to compare it with and other existing median filter (MF) and WFM. We also show the generalization capability and adaptive property of HAF.

II. SYSTEM ARCHITECTURE OF HAF

Here, we will first discuss pre-processing of a 2-D input image. For the problem of interest here, we assume an input gray image sized 256 x 256 with a pixel intensity between 0 and 255. Since a noise-corrupted image contains a high level of uncertainty, we can consider it as an array of fuzzy variables [2][4]. HAF is designed to create three fuzzy membership functions for three fuzzy sets, namely, Dk (Dark), Md (Medium), and Br (Bright). Therefore, each input pixel intensity $p(k, l)$ is considered as a fuzzy variable, and the membership degree of three fuzzy sets, Dk, Md, and Br, are calculated, respectively.

Since fuzzy systems can directly encode structured knowledge in a numerical framework, in order to be easily processed by HAF, the intensity of each input pixel at (k, l) is normalized to $0 \leq p(k, l) \leq 1$. In HAF, a square window of size 3×3 is used to scan across the entire image, where the filter output associated with the centre of the window is denoted as Y . Thus, the elements of the window $W((k, l))$ centered at (k, l) are as follows:

$$\begin{aligned} x_1 &= p(k-1, l-1), x_2 = p(k-1, l), x_3 = p(k-1, l+1), x_4 = p(k, l-1), \\ x_5 &= p(k, l), x_6 = p(k, l+1), x_7 = p(k+1, l-1), x_8 = p(k+1, l), \\ &\text{and } x_9 = p(k+1, l+1). \end{aligned}$$

Each element is considered to be a fuzzy variable, and the membership functions identify the grade of brightness for each input pixel. Equation (1) gives the bell-shaped membership functions used in HAF:

(1)

$$i=1,2 \dots 9, j=D_k, M_d, B_r$$

Where $a_j, b_j,$ and c_j are adjustable parameters of the bell-shaped membership function. Let $p = p(k, l)$ denotes the predicted intensity for a pixel at (k, l) ;

Premise 1:

IF $\{(p(k-1, l-1) \text{ is } D_k) \text{ and } (p(k-1, l) \text{ is } D_k) \text{ and } (p(k-1, l+1) \text{ is } D_k) \text{ and } (p(k, l-1) \text{ is } D_k) \text{ and } (p(k, l) \text{ is } D_k) \text{ and } (p(k, l+1) \text{ is } D_k) \text{ and } (p(k+1, l-1) \text{ is } D_k) \text{ and } (p(k+1, l) \text{ is } D_k) \text{ and } (p(k+1, l+1) \text{ is } D_k)\}$

THEN $\{Y \text{ is } D_k\}$

Premise 2:

IF $\{(p(k-1, l-1) \text{ is } M_d) \text{ and } (p(k-1, l) \text{ is } M_d) \text{ and } (p(k-1, l+1) \text{ is } M_d) \text{ and } (p(k, l-1) \text{ is } M_d) \text{ and } (p(k, l) \text{ is } M_d) \text{ and } (p(k, l+1) \text{ is } M_d) \text{ and } (p(k+1, l-1) \text{ is } M_d) \text{ and } (p(k+1, l) \text{ is } M_d) \text{ and } (p(k+1, l+1) \text{ is } M_d)\}$

THEN $\{Y \text{ is } M_d\}$

Premise 3:

IF $\{(p(k-1, l-1) \text{ is } B_r) \text{ and } (p(k-1, l) \text{ is } B_r) \text{ and } (p(k-1, l+1) \text{ is } B_r) \text{ and } (p(k, l-1) \text{ is } B_r) \text{ and } (p(k, l) \text{ is } B_r) \text{ and } (p(k, l+1) \text{ is } B_r) \text{ and } (p(k+1, l-1) \text{ is } B_r) \text{ and } (p(k+1, l) \text{ is } B_r) \text{ and } (p(k+1, l+1) \text{ is } B_r)\}$

THEN $\{Y \text{ is } B_r\}$

Consequently,

IF $\{p(k, l) \text{ is closest to } D_k\}$ THEN $\{Y \text{ is } D_k\}$ else

IF $\{p(k, l) \text{ is closest to } M_d\}$ THEN $\{Y \text{ is } M_d\}$ else $\{Y \text{ is } B_r\}$

We will first consider a 3×3 window $W(k, l)$ that scans the image from left to right and from top to bottom. In each scan, nine pixels are ranked according to gray intensity.

We further define

$$p(k, l)_i \in N_{imp}$$

If $p(k, l) = \text{Min}\{W(k, l)\}$ or $\text{Max}\{W(k, l)\}$ or $\{p(k, l) \leq T\}$ or $\{p(k, l) \geq (1.0-T)\}$.

Where T is a threshold. The set N_{imp} contains pixels that are most likely corrupted by impulses. To explain, we will use the input sub image as an example.

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$$

Given the nine input pixels x_i of a sub image, HAF performs fuzzification using Eq. (1) i.e., the membership degree $m_{ij} = m_j(x_i)$ is calculated. In particular, the slope (specified by b_j) is made excessively larger than a_j and c_j such that impulsive noise is to be filtered out by the membership functions. Accordingly, in HAF, b_j is initialized as a large number, say, 15. The second step is to normalize the membership degree of each input pixel using

(2)

$$i=1,2 \dots 9, j=D_k, M_d, B_r$$

After that, HAF calculates the weighted input sum

$$sum_j = \sum w_{ij}x_i$$

Finally, in the defuzzification step, the predicted intensity $y=p$ is computed by using

Where

$$x_i = 0 \text{ and } I_i = 0, \text{ if } x_i \in N_{imp},$$

$$x_i = x_i \text{ and } I_i = 1, \text{ otherwise.}$$

The final output of HAF can be determined using the following rule:

If

$$\text{Output } Y = Sum_j \text{ that has minimum } E_j = |p - su,$$

$$j=D_k, M_d, B_r$$

Else

$$\text{Output } Y = p(k,l)$$

End.

That is, if $x_s \in N$, then the three intermediate weighted results Sum_j are compared with the predicted output $p(k, l)$, from among which the one closest to $p(k, l)$ is chosen to replace the pixel intensity at (k, l) . If the pixel

at $p(k,l)_i \in N_p$, then $p(k,l)$ is used as the final output. With this architecture, the performance of HAF hinges on the weights w_{ij} calculated in Step 2, which in turn is determined by the fuzzy membership functions in Step 1.

III. ESTIMATING MEMBERSHIP FUNCTION

Here we present a systematic algorithm, which can be used to obtain a set of membership functions, by which we mean that these functions are ideal for directly performing fuzzification without any training. The proposed approach to obtaining histogram based membership functions(HMF) starts with utilizing the histogram statistics of the corrupted input image to estimate the histogram of the original image. This is different from the approach employed by WFM filter [3]. To facilitate the following discussion, important nomenclatures are first defined as follows:

Membership function of D_k

$$(4)$$

$$i=1,2,\dots,9$$

Membership function of M_d

$$(5)$$

$$i=1,2,\dots,9$$

Membership function of B_r

$$(6)$$

$$i=1,2,\dots,9$$

IV. CONFIGURATION OF MEMBERSHIP FUNCTION USING HISTOGRAM

Given the histogram $H[n]$, define $X(n)$ = normalized input intensity. Also, $b_1=b_2=b_3=15$, as reasoned in the preceding sections. We also denote A as the parameter matrix of HMF:

$$A = \begin{bmatrix} A(1,i) \\ A(2,i) \\ A(3,i) \end{bmatrix}$$

Where

$A(1,i)$ = parameters a_1, b_1, c_1 of membership function D_k ,

$A(2,i)$ = parameters a_2, b_2, c_2 of membership function M_d ,

$A(3,i)$ = parameters a_3, b_3, c_3 of membership function B_r .

Next, we will use the histogram $H[n]$ shown in Fig. 1. (b) to explain how the *initial* parameters of HMF are derived. The histogram is first divided into K equal-length segments, e.g., $K=3$, and we have D_k, M_d , and B_r . We

then define three statistics, *pdf* (potential density function), *Mass*, and *C*, which are useful for describing the intensity features of the divided histogram segment, namely,

$$(7)$$

$$(8)$$

$$(9)$$

In HAF, the values of *Mass* and *C* obtained from the histogram are used as initial values for the parameters *a* and *c*, respectively.

$$(10)$$

$$(11)$$

$$(12)$$

$$C_{D_k}: \text{Centroid of } D_k = \sum_{n \in D_k} (X(n) \times pdf_{D_k}(n))$$

$$C_{M_d}: \text{Centroid of } M_d =$$

$$C_{B_r}: \text{Centroid of } B_r,$$

In particular, *Mass* corresponds to the *support* length for a fuzzy set. It would be expected that the *support* length of M_d would be longer than that of D_k and M_d because more pixels are located at M_d for images generally encountered. After obtaining the initial parameters, we then apply the *conservation in histogram potential* to optimize the membership functions. After tuning is done, the membership function is completely specified because in order to depict the bell-shaped function in Eq. (1), we only need to know the values of parameters *a*, *b*, and *c*.

V. EXPERIMENTS

Based on the use of a Flower image as test input, three experiments are presented which explored the characteristics of HAF. Experiment 1 compared the performance of HAF with MF [1] and WFM. In Experiment 2, we exploited the *generalization* capability and adaptive property of HAF [2] by using three images having similar histogram statistics. As a measure of the objective improvement obtained using the restoration techniques discussed here, we refer to both the input normalized mean square error (*NMSEi*), and the output summed mean square error (*NMSEo*), given by (13) and (14).

$$(13)$$

$$(14)$$

Where T_p is the total number of pixels in the image, X_p represents the pixel intensities of the original (uncorrupted) image, X'_p represents the corrupted input pixels intensities, and Y_p represents the output intensities in the filtered image using HAF. Note that $0 \leq X_p, Y_p, X'_p \leq 1$. The peak signal-to-noise ratio (*PSNR*) for a 256 x 256, 8-bits/pixel images is simply written as

$$(15)$$

$$NMSE = NMSE_o \text{ or } NMSE_i$$

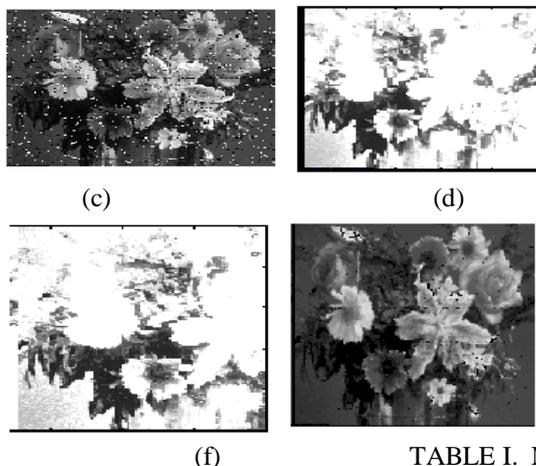
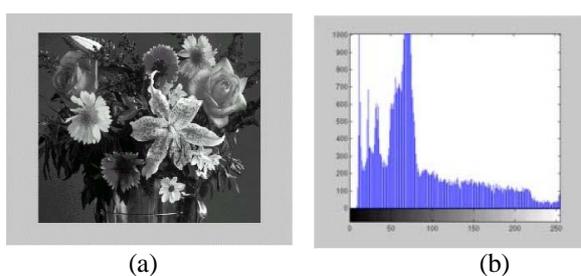
Finally, the improvement in PSNR (*IPSNR*) can be expressed by

IF

1. Experiment 1

Based on the results obtained using the Flower image (256 x 256) as test image, its histogram and corrupted image using salt and pepper noise, HAF output is shown in Fig. 1 (a)-(d)

The performance difference in removing noise using MF, WFM And HAF can be clearly seen by comparing $NMSE_o$ and $PSNR$ between HAF, WFM, and MF are given in Table I , respectively. Clearly, HAF outperforms other filters for $NP=20\%$ all the way up to 90%. Note that MF performs better than WFM and MF performs better than HAF only when $NP \leq 20\%$.

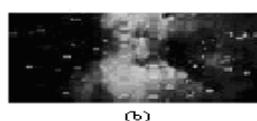
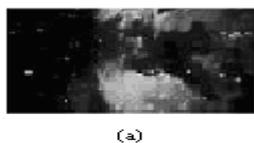


N P	HAF		WFM		MF	
	NM SE _o	PS NR	NM SE _o	PS NR	NM SE _o	PS NR
10 %	0.00 56	22. 51	0.00 65	21. 87	0.00 30	25. 22
20 %	0.00 63	22. 00	0.00 93	20. 31	0.00 41	23. 87
30 %	0.00 74	21. 30	0.01 53	18. 15	0.00 85	20. 70
40 %	0.00 83	20. 80	0.02 61	15. 83	0.01 85	17. 32
50 %	0.00 99	20. 04	0.04 40	13. 56	0.03 66	14. 36
60 %	0.01 30	18. 86	0.07 34	11. 34	0.06 79	11. 68
70 %	0.02 04	16. 90	0.11 74	9.3 0	0.11 46	9.4 0
80 %	0.03 59	14. 44	0.17 80	7.4 9	0.17 70	7.5 2
90 %	0.07 37	11. 32	0.24 62	6.0 8	0.24 74	6.0 6

TABLE I. $NMSE_o$ and $PSNR$ VALUES
Figure 1. (a) original image , (b) Histogram of Flower ,(c) Corrupted by S&P noise , (d) Median Filter(MF) output ,(e) Weighted Fuzzy Median Filter(WFM) output , (f) output of Histogram Adaptive Filter(HAF)

2. Experiment 2

Next, we used the HAF, on pictures that had *similar* histogram statistics. To find out, we used two other ngc4024 pictures denoted as ngc4024s and ngc4024m, respectively as shown if Fig. 2 (a)-(c)





(c)

Figure 1. (a) ngc4024 , (b) ngc4024s ,(c) ngc4024m

$NMSE$ and $PSNR$ results for ngc4024 obtained using HMF are shown in Table II. $NMSE$ and $PSNR$ results for ngc4024s obtained using HMF estimated by using ngc4024 are shown in Table III. Similarly, results for ngc4024m obtained using HMF obtained by using ngc4024 are shown in Table IV.

By comparing Tables II,III and IV, we can see that ngc4024s performs better than both ngc4024m and ngc4024 from $NP=0.1$ to $NP=0.9$. Thus, the results from this experiment confirm that HMF calculated from an arbitrary similar image can be used to restore other images having histogram statistics similar to that of the image. More significantly, the results from Experiment 2 empirically justify use of the HMF.

TABLE II .Comparisons of $NMSE_o$ and $PSNR$ using ngc4024

NP	MF		HAF	
	$NMSE_o$	PSNR	$NMSE_o$	PSNR
10%	0.000623	32.05	0.0171	17.67
20%	0.0012	29.20	0.0251	16.00
30%	0.0049	23.09	0.0308	15.11
50%	0.0392	14.06	0.0357	14.47
70%	0.1262	8.98	0.0501	13.00
90%	0.2770	5.57	0.1106	9.56

NP	MF		HAF	
	$NMSE_o$	PSNR	$NMSE_o$	PSNR
10%	0.000392	34.05	0.0014	28.53
20%	0.000955	30.19	0.0014	28.53
30%	0.0051	22.92	0.0015	28.23
50%	0.0375	14.25	0.0017	27.69
70%	0.1342	8.722	0.0049	23.09
90%	0.2964	5.28	0.0305	15.15

TABLE III.
 $NMSE_o$ and $PSNR$ Comparisons of
using ngc4024sTABLE IV. Comparisons of $NMSE_o$ and $PSNR$ using ngc4024m

NP	MF		HAF	
	$NMSE_o$	PSNR	$NMSE_o$	PSNR
10%	0.000453	33.43	0.0016	27.95
20%	0.0010	30.00	0.0016	27.95
30%	0.0051	22.92	0.0017	27.69
50%	0.0378	14.22	0.0020	26.98
70%	0.1337	8.73	0.0058	22.36
90%	0.2952	5.29	0.0369	14.32

VI. CONCLUSION

A novel adaptive fuzzy filter (HAF), which uses fuzzy spatial filtering optimized via image statistics rather than a priori knowledge of specific image data, has been presented. Instead of using randomly assumed membership functions, an effective algorithm based on input histogram statistics has been proposed to obtain a set of well-conditioned membership functions (i.e., HMF).

For images corrupted by impulsive noise, HAF outperforms MF and WFM filters for the range of impulsive noise probability. Like MF, HAF shows the ability to remove impulsive noise while preserving edge sharpness. We have carried out two experiments to illustrate the effectiveness of HMF and to characterize the restoration power of HAF. In particular, these extensive results verify that there exists a correlation between input histogram statistics and fuzzy membership functions. In this paper, this relationship has been proven useful in

(1) Deriving HMF for HAF to achieve near-optimal noise

Suppression power and

(2) Exploiting the *generalization* capability and adaptive

Property of HAF.

For the later, we have empirically shown that images with similar statistics (in fact, pictures that do not necessarily look similar) can be successfully restored by the HMF inferred from an arbitrary image chosen from these similar images. We believe that the generalization capability and *adaptive* property are useful for making HAF applicable to video transmission where successive image frames must have similar histograms.

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