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The Use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC)

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ABSTRACT
In a reinforced concrete mixture, the cost of the conventional reinforcement constitutes the major cost. Recent researches have shown different ways and techniques of incorporating fibres to partially or wholly replace the conventional reinforcement bar in order to strengthen the economic goal of concrete deign. The use of Asbestos Fibre (AF) as a replacement for conventional reinforcement is one of those new techniques that can be employed under controlled environment. This research study is therefore aimed at using Scheffe's Second Degree Regression Model to optimize the compressive strength of Asbestos Fibre Reinforced Concrete (AFRC). Using Scheffe's Simplex method, the compressive strength of AFRC was evaluated for different mix ratios. Control experiments were simultaneously carried out and the compressive strength determined also. After the tests have been conducted, the adequacy of the model was tested using Student's t-test. The test statistics found the model adequate for predicting the compressive strength of AFRC when the mix ratio is known. Maximum compressive strength for the Scheffe's (5,2) model was obtained as 49.36 MPa. For the fact that structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa as specified by the American Concrete Institute (ACI) and ASTM C 39 or ASTM C 469 respectively, it means that optimized AFRC based on Scheffe's (5,2) model can produce the required compressive strength needed in major construction projects such as construction of walkways, pavement slabs ,building , airports, bridges etc, still maintaining both economic and safety advantages.

Keywords*: AFRC, Scheffe's (5,2) Regression Model, Optimization, Compressive strength ,Mixture Design*

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I. INTRODUCTION

In general, Asbestos is a naturally occurring thin crystalline long fibre. It was first discovered by the Greeks and Romans in the 18th century where they referred to it as a Miracle Mineral. It is obtained by extraction of asbestos-containing rock which is crushed and milled to produce a thread like fibrous material known as asbestos. The asbestos thus obtained contains thousands of fibres which can be further divided into microscopic fibrils. Experience has shown that Asbestos Fibres (AF), when used for the preparation of concrete improves the fresh and hardened properties of concrete such as compressive and flexural strength. In the construction industry, it finds its application in heat and acoustic insulation, fireproofing, roofing and flooring jobs. Other advantages of Asbestos Concrete abound. It is a very good thermal insulator and increases the energy efficiency of the building. It is highly resistant to fire and does not burn easily. It is a very inexpensive and a very cost effective material, hence widely used. It is highly durable and weather-proof. It is resistant to damage from termites. However, some asbestos are classified as hazardous material and hence their applications should only be allowed under controlled environment. AF is added to the concrete in two ways: Addition of asbestos fibre in cement and Addition of asbestos fibre in concrete. This present work deals with the later. Figure 1 shows a typical diagram of an asbestos concrete.

In order to optimize the present concrete mixture, the concept of optimization needs to be explained. Basically, an optimization problem is one requiring the determination of the optimal value of a given function, known as the objective function, subject to a set of stated limitations placed on the concerned variables.it can be observed that in every optimization problem, that there is always need for an objective function which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Specifically for the concrete mixture, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work On the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed, the design of concrete mix according to (Shetty, 2006) has not being a simple task. By definition, concrete mix design according to Jackson and Dhir (1996) remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appears to be a little bit complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Thus, optimization of the concrete mixture design proves to be the fastest method, best option, most convenient and the most efficient way of selecting concrete mix ratios /proportions for better efficiency and better performance of concrete when compared with usual empirical methods .Typical examples of optimization model is Scheffe's Polynomial Model. It could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. In this present study, Scheffe's Second Degree Model for five components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestos Fibre are in focus.

Concrete, according to Oyenuga (2008) is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. Although concrete is one of the most widely used construction material, it has got its own drawbacks. According to Shetty (2006), plain concrete possesses a very low tensile strength, limited ductility, low shear strength and little resistance to cracking. That is, unreinforced (plain) concrete is brittle in nature, and is characterized by low tensile strength but high compressive strength As all stakeholders in the construction industries are focusing on sustainable technology that can be safe and economical, attempts have been made to improve the concrete properties with relatively new construction material developed through extensive research and development work. This has led to the reinforcement of the tension zone of the concrete with conventional steel bars. Due to the expensive nature of the conventional reinforcement, further researches have shown that incorporation of fibres into the concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed. In general, fibres are usually used in concrete to control cracking due to plastic shrinkage and to drying shrinkage. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibres produce greater impact, abrasion, and shatter resistance in concrete. Combining fibres with concrete can produce a range of materials which possess enhanced tensile strength, compressive strength, elasticity, toughness, and durability etc. Asbestos Fibre Reinforced Concrete (AFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is partially or wholly replaced with Asbestos Fibre (AF). Before now, Asbestos was considered as good fibre reinforcement since it was inexpensive, readily available, and easily blended into the mix. Special property of AFRC under investigation in this present work is the compressive strength. It is the strength of hardened concrete measured by the compression test. It is measured by crushing cylindrical concrete specimens in a universal testing machine (UTM). Further, the compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete under investigation.

This present study examines the use of Scheffe's Second Degree Polynomial Model in the optimization of the compressive strength of AFRC. Although, there have been little works done on the general asbestos and optimization applications, none has been able to address the subject matter in full. For example, Chaudhary and others (2017) performed experimental analysis on asbestos fibre reinforcement concrete composite. Samir and others (2014) investigated the use of asbestos –free fibre –cement waste as a partial substitute of Portland cement in mortar. On optimization, recent works have shown that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For instance, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River

Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) .Finally, Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC. Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Finally, Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). From the works reviewed so far, there is enough evidence that the subject matter has not been fully addressed as it can be envisaged that no work has been done on the use of Scheffe's Second Degree Model to optimize the compressive strength of AFRC. Thus, there is urgent need for this present research work.

Fig.1 : A Typical Example Of Asbestos Concrete.

II. BASICS IN SCHEFFE'S OPTIMIZATION MODEL

In this section, important information relating to Scheffe's theory, relevant calculations, prediction of mixture design ratio etc are presented. In general, a simplex lattice in the Scheffe's optimization model is a structural representation of lines joining the atoms of a mixture, where the atoms in turn are constituent components of the mixture. For instance, when considering the AFRC mixture, the relevant constituent elements are the water, cement, fine aggregate, coarse aggregate and the asbestos fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. See Nwachukwu and others (2017). With respect to Scheffe's theory, mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \ldots + X_q = 1 \ ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3 \ldots q$, and q = the number of mixtures

2.1. IMPORTANT INFORMATION ON AFRC SCHEFFE'S (5,2) SIMPLEX LATTICE DESIGN

The Scheffe's (q, m) , e.g Scheffe'(5,2) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen regression equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \ldots, 1; \quad i = 1, 2, \ldots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which is 2 in this present study .For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2 = 6$ points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are : $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. To evaluate the number of coefficients/ or terms/ or design points required for a given lattice , the following general formula is applied:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad {}^{q+m-1}C_m \qquad \mathbf{2(a\text{-}b)}$$

Where k = number of coefficients/ terms / point, q = number of components/mixtures = 5 in this present study
m = number of deqree of polynomial = 2 in this present work. Using either of Eqn. (2), $k_{(5,2)} = 15$
This implies that the possible design points for AFRC Scheffe's (5,2) lattice can be as follows:
A_1 (1,0,0,0,0,); A_2 (0,1,0,0,0,); A_3 (0,0,1,0,0,); A_4 (0,0,0,1,0,), A_5 (0,0,0,0,1,); A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5) **(3)**
According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \ldots X_q$ is given in form of: $Y = b_0 + \sum bi\, xi + \sum bijxj + \sum bi\,jxjxk + + \sum bi\,{}^{j}{}_{2} + \ldots i_n xi_2 xi_n$ **(4)**
where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response (the response is a polynomial function of pseudo component of the mix) which represents the property under study. In this work, the property under study is the compressive strength. This research work is based on the (5, 2) simplex. The actual form of Eqn. (4) has already been developed by Nwachukwu and others (2017) and will be applied subsequently here.

2.2. NOTES ON PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mixture design, the relationship between the pseudo components and the actual components is given as: $Z = A * X$ **(5)**
where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship
Re-arranging the equation, we have : $X = A^{-1} * Z$ **(6)**

2.3. REGRESSION EQUATION FOR AFRC SCHEFFE'S (5, 2) SIMPLEX LATTICE

The polynomial equation by Scheffe (1958), describing the response is given in Eqn.(4). But, for Scheffe's (5,2) simplex lattice, the polynomial equation for five component mixtures has been derived from Eqn.(4) by Nwachukwu and others (2017) and the simplified version is given as follows:

$$Y = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \qquad \mathbf{(7)}$$

2.4 . COEFFICIENTS OF THE AFRC SCHEFFE'S (5, 2) REGRESSION EQUATION

Following the work of Nwachukwu and others (2022h), the coefficients of the Scheffe's (5, 2) regression equation can be expressed as follows. :

$\beta_1 = Y_1$; $\beta_2 = Y_2$; $\beta_3 = Y_3$; $\beta_4 = Y_4$; $\beta_5 = Y_5$; $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$; $\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3$;
8(a-g)

$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4$; $\beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5$; $\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3$; $\beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4$;
9(a-d)

$\beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5$; $\beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4$; $\beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5$; $\beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5$
10(a-d)

Where Y_i = Response Function (or Compressive Strength) for the pure component, i

2.5. SCHEFFE'S (5, 2) MIXTURE DESIGN MODEL FOR AFRC

If we substitute Eqns. (8)-(10) into Eqn. (7), we obtain the mixture design model for the AFRC mixture based on Scheffe's (5, 2) lattice.

2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE AFRC SCHEFFE'S (5,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL POINT AND CONTROL POINT

2.6.1. AT THE INITIAL EXPERIMENTAL TEST POINTS

The requirement of simplex lattice design from Eqn.(1) makes it impossible to use the conventional mix ratios such as 1:2:4, 1:1.3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components (ingredients) proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions were chosen for the five points/vertices.**:**

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8);A_3 (0.5:1:1.2:1.7:1);A_4 (0.7:1:1:1.8:1.2)andA_5 (0.75:1:1.3:1.2:1.5),**(11)** which represent water/cement ratio, cement, fine aggregate, coarse aggregate and asbestos fibre.For the pseudo mix ratio, we have the following corresponding mix ratios at the vertixes:

A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1) **(12)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa , Eqns.(5)and (6) are used..

Substituting the mix ratios from point A_1 into Eqn. (5) gives:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \textbf{(13)}$$

Transforming the R.H matrix and solving , we obtain:

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \qquad \textbf{(14)}$$

Thus

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 . \\ Z_4 \\ Z_5 \end{Bmatrix} \qquad \textbf{(15)}$$

Considering the mix ratios at the midpoints, we have from Eqn. (3):

A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5)

Substituting these pseudo mix ratios in turn into Eqn. (15) will give the corresponding actual mix ratio

For point A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ 1 \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \qquad \textbf{(16)}$$

Hence comparing

$Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$

The rest are shown in Table 1

Hence to generate the polynomial coefficients, fifteen experimental tests will be carried out and the corresponding mix ratio are as depicted in Table 1.

Table 1: Actual Mix Ratios For The AFRC Scheffe's (5, 2) Simplex Lattice

S/N	Points	Water/Cement Ratio (Z_1)	Cement (Z_2)	Fine Aggregate(Z_3)	Coarse Aggregate(Z_4)	Asbestos Fibre(Z_5)	Response Y
1	1	0.67	1	1.70	2.00	0.50	Y_1
2	2	0.56	1	1.60	1.80	0.80	Y_2
3	3	0.50	1	1.20	1.70	1.00	Y_3
4	4	0.70	1	1-00	1.80	1.20	Y_4
5	5	0.75	1	1.30	1.20	1.50	Y_5
6	12	0.62	1	1.65	1.90	0.65	Y_{12}

7	13	0.59	1	1.45	1.85	0.75	Y_{13}
8	14	0.69	1	1.35	1.90	0.85	Y_{14}
9	15	0.71	1	1.50	1.60	1.00	Y_{15}
10	23	0.53	1	1.40	1.75	0.90	Y_{23}
11	24	0.63	1	1.30	1.80	1.00	Y_{24}
12	25	0.66	1	1.45	1.50	1.15	Y_{25}
13	34	0.60	1	1.10	1.75	1.10	Y_{34}
14	35	0.63	1	1.25	1.45	1.25	Y_{35}
15	45	0.73	1	1.15	1.50	1.50	Y_{45}

2.5.2. AT THE EXPERIMENTAL (.CONTROL) POINT

For the purpose of this research, fifteen different controls were predicted which according to Scheffe, their summation should not be more than one.

C_1 = (0.25, 0.25, 0.25, 0.25, 0), C_2 = (0.25, 0.25, 0.25, 0, 0.25), C_3 = (0.25, 0.25, 0, 0.25, 0.25), C_4 = (0.25, 0, 0.25, 0.25, 0.25), C_5 = (0, 0.25, 0.25, 0.25, 0.25), C_{12} = (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} = (0.30, 0.30, 0.30, 0.10, 0), C_{14} = (0.30, 0.30, 0.30, 0, 0.10), C_{15} = (0.30, 0.30, 0, 0.30, 0.1), C_{23} = (0.30, 0, 0.30, 0.30, 0.1), C_{24} = (0, 0.30, 0.30, 0.30, 0.10), C_{25} = (0.10, 0.30, 0.30, 0.30, 0), C_{34} = (0.30, 0.10, 0.30, 0.30, 0), C_{35} = (0.30, 0.30, 0.10, 0.30, 0), C_{45} = (0.10, 0.20, 0.30, 0.40, 0),

Substituting into Eqn.(16) , we obtain the values of the actual mixes as follows:

Control 1 C_1

$$\left\{\begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix}\right\} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \left\{\begin{matrix} 0.61 \\ 1 \\ 1.38 \\ 1.8 \\ 0.5 \end{matrix}\right\} \qquad (17)$$

The rest are shown in Table 2

Table 2: Actual (Z_i) and Pseudo (X_i) Component of AFRC Scheffe's (5, 2) Simplex Lattice At Control Point

S/N	**POINTS**	**PSEUDO COMPONENTS**					**CONTROL POINTS**	**ACTUAL COMPONENTS**				
		Water(X_1)	**Cement (X_1)**	**Fine Aggregate (X_3)**	**Course Aggregate (X_4)**	**Asbestos Fibre(X_5)**		**Water(Z_1)**	**Cement (Z_2)**	**Fine Aggregate (Z_3)**	**Course Aggregate (Z_4)**	**Asbestos Fibre(Z_5)**
1	**1**	0.25	0.25	0.25	0.25	0.00	C_1	0.61	1	1.38	1.83	0.50
2	**2**	0.25	0.25	0.25	0.00	0.25	C_2	0.62	1	1.45	1.68	0.80
3	**3**	0.25	0.25	0.00	0.25	0.25	C_3	0.67	1	1.40	1.70	1.00
4	**4**	0.25	0.00	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.20
5	**5**	0.00	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.50
6	**12**	0.20	0.20	0.20	0.20	0.20	C_{12}	0.64	1	1.36	1.70	0.65
7	**13**	0.30	0.30	0.30	0.10	0.00	C_{13}	0.59	1	1.45	1.83	0.75
8	**14**	0.30	0.30	0.30	0.00	0.10	C_{14}	0.59	1	1.48	1.77	0.85
9	**15**	0.30	0.30	0.00	0.30	0.10	C_{15}	0.65	1	1.42	1.80	1.00
10	**23**	0.30	0.00	0.30	0.30	0.10	C_{23}	0.64	1	1.30	1.77	0.90
11	**24**	0.00	0.30	0.30	0.30	0.10	C_{24}	0.60	1	1.27	1.71	1.00
12	**25**	0.10	0.30	0.30	0.30	0.00	C_{25}	0.60	1	1.31	1.79	1.15
13	**34**	0.30	0.10	0.30	0.30	0.00	C_{34}	0.62	1	1.33	1.83	1.10
14	**35**	0.30	0.30	0.10	0.30	0.00	C_{35}	0.63	1	1.41	1.85	1.25
15	**45**	0.10	0.20	0.30	0.40	0.00	C_{45}	0.61	1	1.25	1.79	0.50

The actual component as transformed from Eqn. (14) , Table (1) and (2) were used to measure out the quantities of Water/Cement Ratio (Z_1), Cement (Z_2), Fine Aggregate (Z_3), Coarse Aggregate (Z_4), and Asbestos Fibre (Z_5) in their respective ratios for the concrete cube strength test.

III. MATERIALS AND METHODS

3.1 MATERIALS

In this present research work, the constituent materials under investigation are Water/Cement ratio, Cement, Fine and Coarse Aggregates and Asbestos Fibre. The water is obtained from potable water from the clean water source. The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). The fine aggregate, with sizes that ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The Asbestos Fibre used in this work is used under controlled environment as raw asbestos, generally is a very hazardous material. Generally, Asbestos is obtained by extraction of asbestos-containing rock which is crushed and milled to produce a thread like fibrous material known as asbestos. The asbestos thus obtained contains thousands of fibres which can be further divided into microscopic fibrils that give rise to Asbestos Fibres (AF).

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimen for the compressive strength is concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28 days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the compressive strength was then calculated using Eqn.(18)

$$\text{Compressive Strength} = \frac{\text{Average failure Load (N)}}{\text{Cross- sectional Area (mm}^2\text{)}} \qquad \frac{P}{A} \qquad \textbf{(18)}$$

IV. RESULTS PRESENTATION AND DISCUSSION

4.1 AFRC RESPONSES FOR THE INITIAL EXPERIMENTAL TEST POINT

The results of the compressive strength (response) test based on Eqn. (18) are shown in Table 3

Table 3: AFRC Compressive Strength (Response) Test Results Based on Eqn.(18)

S/N	**POINTS**	**EXPERIMENT NO**	**RESPONSE Y_I, N/mm^2**	**RESPONSE SYMBOL**	**$\sum Y_I$**	**AVERAGE RESPONSE Y, N/mm^2**
1	1	1A 1B	45.52 46.17	Y_1	91.69	45.85
2	2	2A 2B	42.25 43.10	Y_2	85.35	42.68
3	3	3A 3B	33.48 34.25	Y_3	67.73	33.87
4	4	4A 4B	39.13 40.06	Y_4	79.19	39.60
5	5	5A 5B	44.33 44.84	Y_5	89.17	44.59
6	12	6A 6B	40.23 41.09	Y_{12}	81.32	40.66
7	13	7A 7B	49.38 49.34	Y_{13}	98.72	49.36
8	14	8A 8B	32.49 31.88	Y_{14}	64.37	32.19
9	15	9A 9B	38.24 37.88	Y_{15}	76.12	38.06
10	23	10A 10B	45.87 46.23	Y_{23}	92.10	46.05
11	24	11A 11B	35.12 35.18	Y_{24}	70.30	35.15
12	25	12A 12B	37.75 38.25	Y_{25}	76.00	38.00

13	34	13A 13B	41.84 42.28	Y_{34}	84.12	42.06
14	35	14A 14B	38.45 39.06	Y_{35}	77.51	38.78
15	45	15A 15B	31.27 31.23	Y_{45}	62.50	31.25

4.2. AFRC RESPONSES FOR THE EXPERIMENTAL (CONTROL) TEST POINTS

The response (compressive strength) from experimental (control) tests is shown in Table 4

Table 4: AFRC Response of Control Points from Experimental (control) Tests (5, 2) Simplex Lattice

S/N	POINTS	EXPERIMENT NO	RESPONSE N/mm^2	Z_1	Z_2	Z_3	Z_4	Z_5	AVERAGE RESPONSE		
1	C1	1A 1B	44.92 45.28	0.61	1	1.38	1.83	0.5	44.98	10.42	
2	C2	2A 2B	40.39 39.28	0.62	1	1.45	1.68	0.8	39.84	9.04	
3	C3	3A 3B	32.29 33.10	0.67	1	1.4	1.7	1	32.70	7.33	
4	C4	4A 4B	40.23 39.23	0.66	1	1.3	1.68	1.2	39.73	7.89	
5	C5	5A 5B	43.23 44.24	0.63	1	1.28	1.63	1.5	43.74	12.81	
6	C12	6A 6B	41.23 40.08	0.64	1	1.36	1.7	0.65	40.66	10.77	
7	C13	7A 7B	48.08 48.34	0.59	1	1.45	1.83	0.75	48.31	7.6	
8	C14	8A 8B	30.29 31.11	0.59	1	1.48	1.77	0.85	30.70	8.1	
9	C15	9A 9B	39.31 38.44	0.65	1	1.42	1.8	1	38.88	7.05	
10	C23	10A 10B	46.86 47.18	0.64	1	1.3	1.77	0.9	47.02	7.25	
11	C24	11A 11B	36.24 35.28	0.6	1	1.27	1.71	1	35.76	8.04	
12	C25	12A 12B	38.18 37.25	0.6	1	1.31	1.79	1.15	37.72	7.96	
13	C34	13A 13B	40.82 41.74	0.62	1	1.33	1.83	1.1	41.28	8.14	
14	C35	14A 14B	39.44 40.23	0.63	1	1.41	1.85	1.25	39.84	10.54	
15	C45	15A 15B	32.28 32.42	0.61	1	1.25	1.79	1.35	32.35	11.02	

4.3. SCHEFFE' S (5,2) MODEL FOR THE AFRC RESPONSES.

By substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients of the Scheffe's second degree polynomial for AFRC as follows:

$\beta_1 = 44.98$; $\beta_2 = 39.84$; $\beta_3 = 32.70$; $\beta_4 = 39.73$; $\beta_5 = 43.74$; $\beta_{12} = 162.64$; $\beta_{13} = 37.88$; $\beta_{14} = -46.62$; $\beta_{15}=-21.92$;$\beta_{23}=43$; $\beta_{24} = -16.1.$; $\beta_{25}= -16.28$; $\beta_{34} = 20.26$; $\beta_{35}= 6.48$; $\beta_{45}= -37.54$ **(19)**

Substituting the values of these coefficients in Eqn.(19) into Eqn. (9) yield the regression model for the optimization of the compressive strength of AFRC based on Scheffe's (5,2) lattice as given in Eqn.(20)

$$Y = 44.98X_1 + 39.84X_2 + 32.70X_3 + 39.73X_4 + 43.74X_5 + 162.64\,X_1X_2 + 37.88X_1X_3 - 46.62X_1X_4 - 21.92X_1X_5 + 43.00X_2X_3 - 16.1X_2X_4 - 16.28\,X_2X_5 + 20.26X_3X_4 + 6.48X_3X_5 - 37.54X_4X_5 \qquad \textbf{(20)}$$

4.4. SCHEFFE'S (5,2) MODEL RESPONSES FOR AFRC AT CONTROL POINTS

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 2 into Eqn.(20) yields the second degree model responses for the control points of AFRC.

4.5.VALIDATION OF AFRC MODEL RESULTS USING STUDENT'S – T -TEST

In order to check the correlation between the compressive strength results (lab responses) given in Table 4 and model responses from the control points based on Eqn.(20), we perform the test of adequacy using the Student's – T – test . The procedures/steps involved for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model responses. Thus, the Scheffe's model is adequate for predicting the compressive strength of AFRC based on Scheffe's (5,2) lattice.

4.6. DISCUSSION OF RESULTS

The maximum compressive strength of AFRC based on Scheffe's (5,2) lattice is 49.36MPa . This corresponds to mix ratio of 0.59:1.00:1.45:1.85:0.75 for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestor Fibre respectively. The minimum compressive strength is 31.25 MPa which also correspond to the mix ratio of 0.73:1.00:1.15:1.50: 1.50 for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestos Fibre respectively. From the results, the maximum value from the Scheffe's model is greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete. Thus, the Scheffe's model can be used to determine the AFRC compressive strength of all points (1 - 45) in the simplex based on Scheffe's Second Degree Model**.**

V. CONCLUSION

In this work so far, Scheffe's Second Degree Regression Model was used to formulate a model for predicting the compressive strength of AFRC cubes. In the first instance, the Scheffe's method was used to predict the mix ratio for evaluating the compressive strength of AFRC. By using Scheffe's (5,2) simplex model, the values of the compressive strength were determined at all 15 points (1- 45). The results of the student's t-test validated the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength predicted by the model based on Scheffe's (5,2) model is 49.36MPa. As expected, the maximum value meets the minimum standard requirement of 20 MPa and 30.75MPa stipulated by American Concrete Institute (ACI) and ASTM C 39 or ASTM C 469 respectively for the compressive strength of good and high performance concrete. Thus, with the Scheffe's (5,2) model for AFRC any desired strength, given any mix proportions can be easily predicted and evaluated and vice versa. Therefore, the utilization of this Scheffe's optimization model has solved the problem of having to go through vigorous, time-consuming and laborious mixture design procedures in other to obtain the desired strength.

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