

Application of the Optimization Problem to Select Oil Flow from Wells in the Oil Industry

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Abstract: In recent years, optimization methods have been widely and effectively applied in economics, engineering, transportation, information technology, and many other scientific disciplines. This article will systematize a real-life model that needs the help of Mathematics to solve production cost problems in business to clarify the relationship between Mathematics and practice

Keywords: Optimization, Optimal, Mathematical model, Oil industry.

Date of Submission: 05-05-2022

Date of Acceptance: 19-05-2022

I. INTRODUCTION

Currently, the direction of scientific research to serve practice and solve problems arising in practice is of great interest. One of the most interested directions is the field of mathematical optimization [1-25]. When planning production and design based on optimization principles, it will save costs in terms of capital, raw materials, time and labor while increasing efficiency, productivity and quality of work. Therefore, the problem is to model real-world problems into optimization problems. Then, the results of the optimization problem will give us the most reasonable production plan in practice.

Currently, oil exploration and production in the world in general and in Vietnam in particular is being conducted very urgently. To speed up the exploration, the support of science and technology is very necessary. Linear programming also plays a big part in this industry. Linear programming makes project establishment and analysis of oil production more accurate and faster. Let's look at some specific examples of the application of Linear Programming in this industry.

II. APPLICATION OF LINEAR PROGRAMMING

To exploit oil fields, people often use the flooding method, in which the water injection must ensure the injection conditions of the borehole. The oil extracted from those boreholes requires final processing in the water separators. Therefore, there is a need to ensure the selection of oil between drilling holes so that the proposed plan can be implemented at the lowest cost but still meets the technical constraints due to the physical properties of the material liquid, of the saturated stratum and extraction method. In this example, we will build a mathematical model for this problem.

We consider n wells. Put $x_j^{(n)}$ and $x_j^{(d)}$ are the amount of water and the amount of oil in the total flow of the j -th well ($j = 1..n$). (the total flow of the j -th well is $x_j = x_j^{(n)} + x_j^{(d)}$)

The score $\alpha_j = \frac{x_j^{(n)}}{x_j}$ is called the inundation coefficient, characterize each well and assuming we know this score.

In order for a well to operate normally, the following conditions must be satisfied:

- 1) At the i -th well, the well bottom pressure p_i always greater than or equal to some predetermined pressure p_i^* (p_i^* is called saturation pressure or vapor intake pressure).
- 2) So that the well can inject, the dynamic pressure q_i must exceed a fixed value q_i^* .

When the fluid movement depends on the linear filter law and in the water pressure regime of the stage, agents p_i and q_i are linear functions of the total flow of the wells x_j . It means:

$$p_i = \sum_{j=1}^n a_{ij}x_j, \quad q_i = \sum_{j=1}^n b_{ij}x_j, \quad i = 1..n$$

The constants a_{ij} , b_{ij} depend on specific conditions such as the mutual distribution between the wells, the characteristics of the stratigraphy and the extraction method... Then, the total amount of dewatered oil is:

$$Q = \sum_{j=1}^n x_j^{(d)} = \sum_{j=1}^n (x_j - x_j^{(n)}) = \sum_{j=1}^n \left(1 - \frac{x_j^{(n)}}{x_j}\right) x_j = \sum_{j=1}^n (1 - \alpha_j) x_j$$

The cost of mining by flooding is denoted by S . This cost can be divided into 2 parts:

+ Part I is the cost of equipment, labor, overhaul..., denoted by S_1 with $S_1 = \sum_{j=1}^n S_{1j}$, in this S_{1j} is the

cost of the I-th group for the j-th well. This part does not depend on mining traffic.

+ Part II depends on the total extraction flow and the amount of water, the amount of oil in the total production flow, denoted by S_2 with

$$S_2 = \sum_{j=1}^n S_{2j} = \sum_{j=1}^n (a_j x_j + b_j x_j^{(n)} + c_j x_j^{(d)}),$$

At the j-th well, a_j is the cost of resisting stratigraphic pressure, b_j is the cost of wastewater treatment, c_j is the cost for processing and transferring oil. These cost factors are calculated per $1m^3$ of the respective liquid.

In fact, we also have to consider the cost of petroleum water separation. This cost is linearly dependent on the quantities $x_j^{(n)}$ and $x_j^{(d)}$, so we can calculate this cost together with the cost of mining by varying the coefficients b_j, c_j .

So the total cost of mining is

$$S = S_1 + S_2 = S_1 + \sum_{j=1}^n (a_j x_j + b_j x_j^{(n)} + c_j x_j^{(d)})$$

$$\text{But } a_j x_j + b_j x_j^{(n)} + c_j x_j^{(d)} = \left(a_j + b_j \frac{x_j^{(n)}}{x_j} + c_j \frac{x_j^{(d)}}{x_j} \right) x_j = (a_j + b_j \alpha_j + c_j (1 - \alpha_j)) x_j$$

so we should be able to write: $S = S_1 + S_2 = S_1 + \sum_{j=1}^n r_j x_j$, $r_j = a_j + b_j \alpha_j + c_j (1 - \alpha_j)$.

In fact, we need to find the optimal way to extract oil from oil wells with the aim of having the lowest cost. However, mining needs to satisfy some requirements: The total amount of dewatered oil extracted must be at least is Q^* (Q^* is predetermined), the conditions of well bottom pressure and dynamic pressure must be satisfied for the well to be able to operate normally (these conditions have been mentioned above). Accordingly, we have the following linear programming problem:

$$S_2 = S - S_1 = \sum_{j=1}^n r_j x_j \rightarrow \text{min}$$

$$\text{with the system of constraints : } \begin{cases} \sum_{j=1}^n (1 - \alpha_j) x_j \geq Q^* \\ \sum_{j=1}^n a_{ij} x_j \geq p_i^*, i = 1..n \\ \sum_{j=1}^n b_{ij} x_j \geq q_i^*, i = 1..n \\ x_j \geq 0, j = 1..n \end{cases}$$

Solving this problem, we will find the optimal solution to determine the flow of oil to be exploited in n wells to achieve the specified economic purpose.

In fact, if we have predetermined costs for mining and oil processing, we can pose a different problem: determine the production flow in the wells so that the maximum amount of dewatered oil can be obtained with the assumed cost not exceeding a given S_2^* and still meeting the technical requirements. Then the mathematical model of this problem is:

$$Q = \sum_{j=1}^n (1 - \alpha_j) x_j \rightarrow \text{max}$$

with the system of constraints:

$$\begin{cases} \sum_{j=1}^n r_j x_j \leq S_2^* \\ \sum_{j=1}^n a_{ij} x_j \geq p_i^*, i = 1..n \\ \sum_{j=1}^n b_{ij} x_j \geq q_i^*, i = 1..n \\ x_j \geq 0, j = 1..n \end{cases}$$

III. CONCLUSION

The paper presents a method application of linear programming to select oil flow from wells in the oil industry. This further elucidates the two-way intimate relationship between mathematics and practice and the important role of mathematics in practice. In the future, the author's follow-up studies will carry out research on optimization for the processes occurring in the engineering process.

Conflict of interest

There is no conflict to disclose.

ACKNOWLEDGEMENT

This work was supported by Thai Nguyen University of Technology, Vietnam.

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