

# An Analysis on Minutes of the Goals in Football Matches with the Nonhomogeneous Poisson Processes

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## Abstract

This study analyses the counts of the goals scored by Galatasaray, Fenerbahçe, Beşiktaş, and Trabzonspor between the minutes 0-15, 16-30, 31-45, 46-60, 61-75, and 76-90 in Turkey during Super League between 2013-2014 season and 2021 season. The probability functions were developed accordingly. By analyzing the goal counts with the management of the nonhomogeneous Poisson processes, teams' probability of scoring 0, 1, 2 goals in each time frame was calculated. As a result, it is seen that the teams' probability of scoring a goal with the nonhomogeneous Poisson processes with respect to time periods and the probability of two combined Poisson teams scoring goals can be calculated.

**Keywords:** Poisson process, probability functions, goal

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## I. INTRODUCTION

Spor Toto Super League, is the highest ranked football league in Turkey. In the league where 21 teams compete in one season, the double-circuit league procedure is applied as each team plays two matches with each other. While the team ranks the top becomes a champion, the 4 teams at the bottom are moved to the 1st League, and the three teams in the 1st League advance to Super League to compete the following season. 2020-21 season of Super League lasting nine months a year consists of 40 weeks and 420 games. By the completion of 2018-19 season, the league ranks 10th in the UEFA Country Ranking and sends 2 teams to the UEFA Champions League and 3 teams to the UEFA Europa League. A team becoming the champion of the Turkish Cup but not ranking among the top 4 can attend the UEFA Europe League.

6 teams have won championships in the Super League where 73 teams competed until today. These teams are Galatasaray (22), Fenerbahçe (19), Beşiktaş (16), Trabzonspor (6), Bursaspor (1) and İstanbul Başakşehir (1). Lastly, in the 2020-21 season Beşiktaş has won the championship (Anonym, 2021a).

The aim in football is to score a goal and football is a goal game. The teams aim to score more goals and won the championship. A football game lasts 90 minutes and measuring the teams' success may be required at various points or time frames during the game. In each 15 minutes of time frame, teams may score different number of goals and perform differently. It is expected that the goal counts in each time frame is according to the Poisson distribution. Poisson processes are applied to the events that show the Poisson distribution.

There have been studies in various fields conducted on Poisson processes. Uzgören and Eleveli (2010) have found the failure count expected in the reliability analysis of a mining machine with the nonhomogeneous Poisson processes, the next expected failure time, and the reliability values for different time frames. In another study, the probability of destructive earthquakes with the intensity of 6.0, 6.5 and 7.0 occurring in the province of Çankırı, Turkey over the 100 years was calculated with the Poisson method as 78%, 45%, and 26% respectively (Köle, 2016). An implementation on traffic accidents was performed with first stage of the combined Poisson distribution (Özel and İnal, 2011). The estimation of shift point was examined with the Poisson process (Yiğiter and İnal, 2007).

The aim of the study is to determine the probability of scoring a goal of the strongest 4 teams competing in Super League Turkey during specific time frames.

## II. MATERIAL AND METHOD

### Material

In the present study, the probability of scoring 0,1,2...goal in the first 15 minutes and between the minutes 16-30, 31-45, 46-60, 61-75 or 76-90 by using only the goals scored and by keeping other factors (weather, field ground, missing player, audience, a match in the field of one team) constant is calculated. Furthermore, the goals to be scored by teams A and B competing against each other are calculated. The minutes which are intended to be estimated during the 90 minute of the match played by any two teams selected are

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acquired using the methods of nonhomogeneous Poisson and common Poisson processes. The results overlap in both of the studies.

The data used in the study consists of goal counts scored between the years 2013-14 and 2020-21 in Turkey during the Spor Toto Super League by four big teams that have won the championship, Galatasaray, Fenerbahçe, Beşiktaş, and Trabzonspor. Information on the goal minutes belonging to 8 seasons in total was obtained each week and recorded. The information gathered from the following website (Anonym, 2021b):

“[\*\*Table 1.\*\* The goal distribution of Galatasaray throughout the seasons](https://www.google.com/search?q=spor+toto+s%C3%BCper+lig&sxsrf=ALeKk03sSE0UG_iltSg5QSnB0ZqFJbD8Cg%3A1621510383688&source=hp&ei=70imYPrVJ4iulwTRzICIBA&iflsig=AINFCbYAAAAAYKZW_zNO6EF4Rhf7dNhdDvzK3_K8OvQ&gs_ssp=eJzj4tPITcwKbK0KDJg9BIpLsgvUijJL8lXKD68pyC1SCEnMx0Ar1sLhg&oq=spor&gs_lcp=Cgndnd3Mtd2l6EAMYADIHCC4QJxCTAjIECCMQJzICAAyAggAMgIIADICCAyAggAMgIIADICCAA6AgguUO3EAlihygJgjNYCaABwAHgAgAGuA4gB9wiSAQkwLjluMC4xLjGYAQCgAQGqAQdnd3Mtd2l6&scient=gws-wiz”</a> These goals are in 6 divisions 0-15, 16-30, 31-45, 46-60, 61-75, 76-90.</p>
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Galatasaray	Minutes					
	0-15	16-30	31-45	46-60	61-75	76-90
Years (season)						
2013-2014	12	6	9	13	12	7
2014-2015	6	12	5	13	9	16
2015-2016	10	3	9	16	12	19
2016-2017	12	8	13	10	4	18
2017-2018	12	12	15	10	8	18
2018-2019	3	13	11	17	14	14
2019-2020	6	11	13	5	7	13
2020-2021	11	8	18	9	12	22
<b>Total</b>	<b>72</b>	<b>73</b>	<b>93</b>	<b>93</b>	<b>78</b>	<b>127</b>

**Table 2.** The goal distribution of Fenerbahçe throughout the seasons

Fenerbahçe	Minutes					
	0-15	16-30	31-45	46-60	61-75	76-90
Years (season)						
2013-2014	10	7	15	8	12	22
2014-2015	13	3	8	12	9	17
2015-2016	6	8	16	9	8	13
2016-2017	4	13	14	7	6	16
2017-2018	11	11	13	11	16	16
2018-2019	4	10	6	3	14	7
2019-2020	6	11	10	6	7	18
2020-2021	8	9	16	12	9	18
<b>Total</b>	<b>62</b>	<b>72</b>	<b>98</b>	<b>68</b>	<b>81</b>	<b>127</b>

**Table 3.** The goal distribution of Beşiktaş throughout the seasons

Beşiktaş	Minutes					
	0-15	16-30	31-45	46-60	61-75	76-90
Years (season)						
2013-2014	7	9	14	6	10	7
2014-2015	3	9	8	10	9	16
2015-2016	6	13	11	11	17	17
2016-2017	12	8	12	13	12	16
2017-2018	9	10	8	13	9	20
2018-2019	9	4	19	12	9	19

2019-2020	6	3	8	12	14	16
2020-2021	10	8	18	14	20	19
<b>Total</b>	<b>62</b>	<b>64</b>	<b>98</b>	<b>91</b>	<b>100</b>	<b>130</b>

**Table 4.** The goal distribution of Trabzonspor throughout the seasons

Trabzonspor	Minutes					
Years (season)	0-15	16-30	31-45	46-60	61-75	76-90
2013-2014	9	7	7	11	10	9
2014-2015	4	11	9	8	9	17
2015-2016	7	3	6	8	7	9
2016-2017	4	8	6	10	7	4
2017-2018	10	3	9	11	9	20
2018-2019	5	15	7	7	13	17
2019-2020	16	9	11	10	13	17
2020-2021	3	11	9	8	9	10
<b>Total</b>	<b>58</b>	<b>67</b>	<b>64</b>	<b>73</b>	<b>77</b>	<b>103</b>

### III. METHOD

To see the event count occurring in the time frame of  $N_t$  (0,t], the nonhomogeneous Poisson process showcased with  $\{N_t, t \geq 0\}$  provides the following axioms (Kadilar, 2020).

- $N_0=0$  and  $P(N_0=0)=1$
- Regarding  $0 \leq t_1 \leq t_2 \leq \dots \leq t_m$ , random variables of  $N_{t_1}, N_{t_2} - N_{t_1}, \dots, N_{t_m} - N_{t_{m-1}}$  is independent. Briefly, the nonhomogeneous Poisson process is a process with independent increments.
- $N_t$  for  $t>0$  is a random variable with  $\Lambda_t = \int_0^t \lambda(u)du$  averaged and Poisson distribution. the probability function of  $N_t$  is as follows:

$$P(N_t = i) = \begin{cases} e^{-\Lambda_t} \frac{(\Lambda_t)^i}{i!}, & i = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

Let  $\{N_t, t \geq 0\}$  be a homogeneous or nonhomogeneous Poisson process and let  $Y_i, i=1,2,3,\dots$  be independent and identically distributed random variables, independent of the process  $\{N_t, t \geq 0\}$ . A stochastic process  $\{X_t, t \geq 0\}$  is said to be a compound Poisson process if it can be displayed as

$$X_t = \sum_{i=1}^{N_t} Y_i$$

If  $E(Y_i) = \eta$  and  $V(Y_i) = \sigma^2, i=1,2,3,\dots$ , expected value and variance of  $X_t$  are  $E(X_t) = \lambda t \eta$  and  $V(X_t) = \lambda t (\sigma^2 + \eta^2)$ , respectively. In particular, if  $Y_i, i=1,2,3,\dots$ , are distribute according to the binomial distribution  $\{X_t, t \geq 0\}$  is called as a Neyman type B process (Ozel and Inal, 2008).

A compound Poisson process S has the following properties (Daniel, 2019).

- For  $t>0, S(t) = \sum_{i=1}^{N(t)} X_i$
- N is a Poisson process with rate function  $\lambda$
- All the random variables  $X_i$  have the same distribution as a single random variable X
- For all t, the random variables N(t) and all the  $X_i$  form an independent set
- If  $N(t)=0$  for a particular value of t, then the empty  $\sum S(t) = 0$  is taken, so in particular,  $S(0)=0$ .

### IV. RESULTS

The average goal counts are going to be the  $\lambda_1(t)$  and  $\lambda_2(t)$  variables for periods of 15 minutes each between Table 1 and 4. These variables are found by counting each season (year) as a series of 34 matches and dividing the average of each period by 34. Each team played 40 matches in the 2020-2021 season while in others they played 34 each. In fact, teams have played  $7*34+40=278$  matches in total throughout these 8 seasons. In other words,  $\lambda(t)$  variables were calculated by dividing the total goals of 8 seasons by 278 in

each period of 15 minutes. It could be noted as the averaging of the mean. The goal counts of teams scored in the periods of 15 minutes namely  $\lambda(t)$  are presented in Table 5.

**Table 5.** Scoring speed of goals in the time frames of 15 minutes

Teams	0-15	16-30	31-45	46-60	61-75	76-90
<b>Galatasaray</b>	0.258993	0.26259	0.334532	0.334532	0.280576	0.456835
<b>Fenerbahçe</b>	0.223022	0.258993	0.352518	0.244604	0.291367	0.456835
<b>Beşiktaş</b>	0.223022	0.230216	0.352518	0.327338	0.359712	0.467626
<b>Trabzonspor</b>	0.208633	0.241007	0.230216	0.26259	0.276978	0.370504

As a result of these calculations, nonhomogeneous Poisson processes are going to be used to calculate the probability of Galatasaray scoring 1 goal between minutes 0-15. Considering that a football match lasts 90 minutes and  $t$  variable is the interval  $[0,90]$ .

In the interval  $[0,15]$ , the scoring speed of goals is going to be 0.258993 goals per match.  $0.258993/15=0.017266$  minutes per match will be required here. The following probability functions are acquired if other periods are calculated accordingly. Regarding  $\lambda_1$  for Galatasaray,  $\lambda_2$  for Fenerbahçe,  $\lambda_3$  for Beşiktaş and  $\lambda_4$  for Trabzonspor, following calculations namely Poisson probability functions can be written.

$$\lambda_1(t) = \begin{cases} 0.017266 \rightarrow 0 \leq t \leq 15 \\ 0.017506 \rightarrow 16 \leq t \leq 30 \\ 0.022302 \rightarrow 31 \leq t \leq 45 \\ 0.022302 \rightarrow 46 \leq t \leq 60 \\ 0.018705 \rightarrow 61 \leq t \leq 75 \\ 0.030456 \rightarrow 76 \leq t \leq 90 \end{cases}$$

$$\lambda_2(t) = \begin{cases} 0.014868 \rightarrow 0 \leq t \leq 15 \\ 0.017266 \rightarrow 16 \leq t \leq 30 \\ 0.023501 \rightarrow 31 \leq t \leq 45 \\ 0.016307 \rightarrow 46 \leq t \leq 60 \\ 0.019424 \rightarrow 61 \leq t \leq 75 \\ 0.030456 \rightarrow 76 \leq t \leq 90 \end{cases}$$

$$\lambda_3(t) = \begin{cases} 0.014868 \rightarrow 0 \leq t \leq 15 \\ 0.015348 \rightarrow 16 \leq t \leq 30 \\ 0.023501 \rightarrow 31 \leq t \leq 45 \\ 0.021823 \rightarrow 46 \leq t \leq 60 \\ 0.023981 \rightarrow 61 \leq t \leq 75 \\ 0.031175 \rightarrow 76 \leq t \leq 90 \end{cases}$$

$$\lambda_4(t) = \begin{cases} 0.013909 \rightarrow 0 \leq t \leq 15 \\ 0.016067 \rightarrow 16 \leq t \leq 30 \\ 0.015348 \rightarrow 31 \leq t \leq 45 \\ 0.017506 \rightarrow 46 \leq t \leq 60 \\ 0.018465 \rightarrow 61 \leq t \leq 75 \\ 0.0247 \rightarrow 76 \leq t \leq 90 \end{cases}$$

The graphic of probability function is presented in Figure 1.

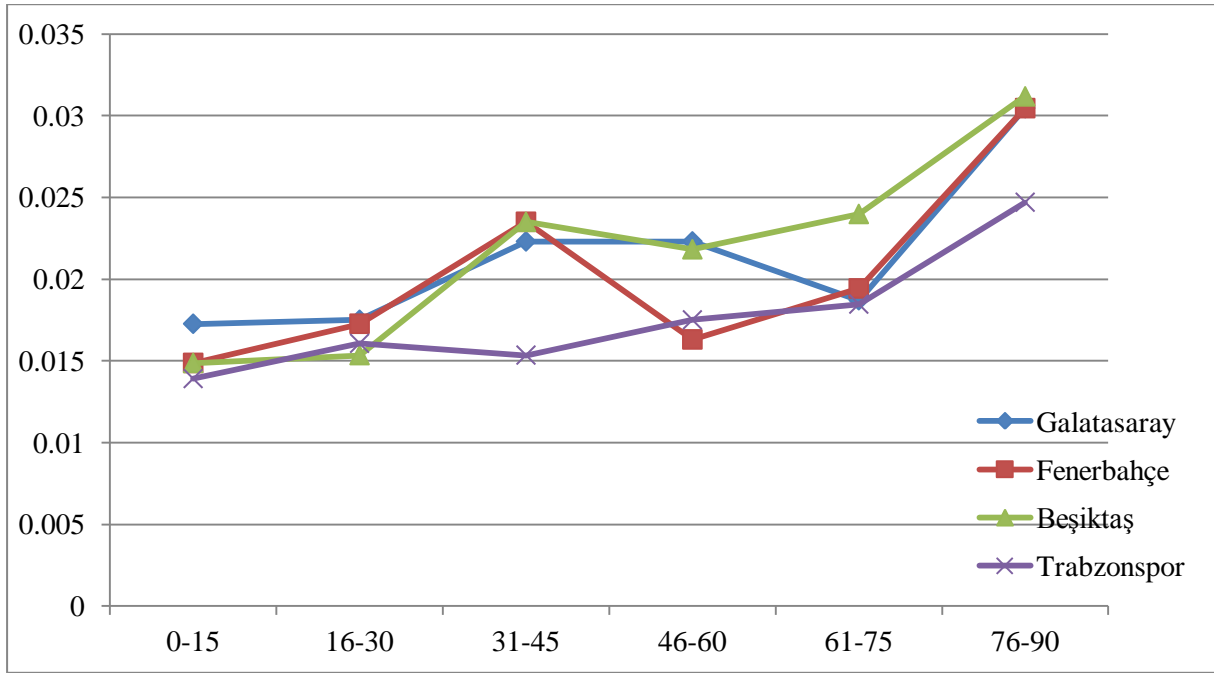


Figure 1. Function graphic

With the nonhomogeneous Poisson method, find the teams' probability of scoring 1 goal in the first 15 minutes.

$$P(N_{15} - N_0 = 1) = \frac{e^{-[m(15)-m(0)]}[m(15) - m(0)]^1}{1!}$$

In the equation above,  $[m(15) - m(0)]$  equation shows the average goal count acquired in the interval  $[0,15]$ . This equation found as:

$$[m(15) - m(0)] = \int_0^{15} \lambda_1(t)dt = \int_0^{15} 0.017266 dt = 0.258993$$

$$P(N_{15} - N_0 = 1) = \frac{e^{-0.258993}(0.258993)^1}{1!} = 0.2$$

This signifies that Galatasaray's probability of scoring a goal in the first 15 minutes is 0.2 which is 20%. Similarly, in the interval  $[0,15]$ , if the following calculations are performed in order for Fenerbahçe to score 2 goals.

$$[m(15) - m(0)] = \int_0^{15} \lambda_2(t)dt = \int_0^{15} 0.014868dt = 0.223022$$

$$P(N_{15} - N_0 = 2) = \frac{e^{-0.223022}(0.223022)^2}{2!} = 0.02$$

Fenerbahçe's probability of scoring 2 goals in the first 15 minutes is 0.02 which is 2%. When the probability of Beşiktaş scoring 1 or 2 goals in the  $[0,15]$  interval is calculated separately,

$$[m(15) - m(0)] = \int_0^{15} \lambda_3(t)dt = \int_0^{15} 0.014868dt = 0.223022$$

$$P(N_{15} - N_0 = 1) = \frac{e^{-0.223022}(0.223022)^1}{1!} = 0.178439$$

Beşiktaş's probability of scoring 1 goal in the first 15 minutes is 0.178439 which is approximately 17.8%.

$$P(N_{15} - N_0 = 2) = \frac{e^{-0.223022}(0.223022)^2}{2!} = 0.02$$

Beşiktaş's probability of scoring 2 goals in the first 15 minutes is 0.02 (2%).

When the probability of Trabzonspor scoring 1 goal in the [0,15] interval is calculated,

$$[m(15) - m(0)] = \int_0^{15} \lambda_4(t)dt = \int_0^{15} 0.013909dt = 0.208633$$

$$P(N_{15} - N_0 = 1) = \frac{e^{-0.208633}(0.208633)^1}{1!} = 0.169346$$

Trabzonspor's probability of scoring 1 goal in the first 15 minutes is 0.169346 which is approximately 16.93%. If the probability of Beşiktaş scoring 2 or 3 goals in the interval [30-45] is calculated,

$$[m(45) - m(30)] = \int_{30}^{45} \lambda_3(t)dt = \int_{30}^{45} 0.023501dt = 0.352518$$

$$P(N_{45} - N_{30} = 2) = \frac{e^{-0.352518}(0.352518)^2}{2!} = 0.043675$$

$$P(N_{45} - N_{30} = 3) = \frac{e^{-0.352518}(0.352518)^3}{3!} = 0.005132$$

Beşiktaş's probability of scoring 2 goals in the interval [30-45] is 0.043675 and for 3 goals it is 0.005132. The probabilities of scoring 1 goal in the time intervals [0-15], [16-30], [31-45], [46-60], [61-75], [76-90] were calculated for 4 teams and presented in Table 6.

**Table 6.** Probabilities of teams scoring 1 goal in time periods

Period /Teams	Galatasaray	Fenerbahçe	Beşiktaş	Trabzonspor
<b>0-15</b>	0.199898	0.178439	0.178439	0.169346
<b>16-30</b>	0.201947	0.199898	0.182875	0.189392
<b>31-45</b>	0.239416	0.247791	0.247791	0.182875
<b>46-60</b>	0.239416	0.191529	0.235958	0.201947
<b>61-75</b>	0.211932	0.217721	0.251035	0.209969
<b>76-90</b>	0.289307	0.289307	0.292962	0.255791

When Table 6 is examined, it can be seen that the probability of 4 big teams scoring at least 1 goal in the first 15 minutes of the match in the Turkish Super League is in the interval [0-15]. It has been observed that the probability of scoring at most 1 goal in the last 15 minutes is in the interval [76-90]. In [76-90] time period, Beşiktaş has the highest probability of scoring 1 goal. The probability of Beşiktaş scoring 1 goal in the last 15 minutes of the match in this time period is approximately 0.293 which is 29.3%. It is followed by Galatasaray and Fenerbahçe. The probability of both teams to score 1 goal in this time period is equal to 0.289. The probability of Trabzonspor scoring 1 goal in the same time period is 0.256.

If the probability of any two teams scoring the same or different number of goals together in a specific time interval is calculated, the resultant functions is going to be the multiplication of one other according to the Poisson process since these two processes are independent. In that case, the probability of Beşiktaş team scoring 1 goal and Trabzonspor team scoring 2 goals in the interval [0,15] is calculated as follows.  
 $P(\text{Beşiktaş}=1, \text{Trabzon}=2)=0.178439*0.017666=0.003152$

Similar to this calculation, the probability of scoring goals among teams in different time intervals has been calculated and presented in Table 7.

**Table 7.** The teams' probability of scoring goals in the mutually required time interval.

Teams	Goal	0-15	16-30	31-45	46-60	61-75	76-90
Beşiktaş	1	0.17844	0.18287	0.24779	0.23596	0.25103	0.29296
Trabzon	0	0.81169	0.78584	0.79436	0.76906	0.75807	0.69039
	Two teams	<b>0.14484</b>	<b>0.14371</b>	<b>0.19684</b>	<b>0.18147</b>	<b>0.19030</b>	<b>0.20226</b>
Beşiktaş	1	0.17844	0.18287	0.24779	0.23596	0.25103	0.29296
Trabzon	1	0.16935	0.18939	0.18287	0.20195	0.20997	0.25579
	Two teams	<b>0.03022</b>	<b>0.03464</b>	<b>0.04531</b>	<b>0.04765</b>	<b>0.05271</b>	<b>0.07494</b>

Beşiktaş	1	0.17844	0.18287	0.24779	0.23596	0.25103	0.29296
Galatasaray	1	0.19990	0.20195	0.23942	0.23942	0.21193	0.28931
	Two teams	<b>0.03567</b>	<b>0.03693</b>	<b>0.05932</b>	<b>0.05649</b>	<b>0.05320</b>	<b>0.08476</b>
Beşiktaş	2	0.01990	0.02105	0.04368	0.03862	0.04515	0.06850
Galatasaray	1	0.19990	0.20195	0.23942	0.23942	0.21193	0.28931
	Two teams	<b>0.00398</b>	<b>0.00425</b>	<b>0.01046</b>	<b>0.00925</b>	<b>0.00957</b>	<b>0.01982</b>
Beşiktaş	1	0.17844	0.18287	0.24779	0.23596	0.25103	0.29296
Fenerbahçe	0	0.80010	0.77183	0.70292	0.78301	0.74724	0.63329
	Two teams	<b>0.14277</b>	<b>0.14115</b>	<b>0.17418</b>	<b>0.18476</b>	<b>0.18758</b>	<b>0.18553</b>
Beşiktaş	2	0.01990	0.02105	0.04368	0.03862	0.04515	0.06850
Fenerbahçe	1	0.17844	0.19990	0.24779	0.19153	0.21772	0.28931
	Two teams	<b>0.00355</b>	<b>0.00421</b>	<b>0.01082</b>	<b>0.00740</b>	<b>0.00983</b>	<b>0.01982</b>

As can be seen in Table 7, Beşiktaş which has won the championship in the last season, Galatasaray, Fenerbahçe and Trabzonspor hold the probability of scoring goals amongst one another as two teams each in a specific time period. For example, the probability of Beşiktaş and Trabzonspor scoring 1-1 in the time interval [31-45] is 0.04531. Accordingly, the probability of Beşiktaş scoring 1 goal and Trabzonspor 0 goals in a match is between 14.37% and 20.23%. During a match between Beşiktaş and Trabzonspor, the probability of the teams scoring 1 goal in different time periods is between 3.02% and 7.49%. The probability of a match between Beşiktaş-Galatasaray with a score of 1-1 in the first 15 minutes is 3.57%, and the probability of the score being 2-1 and in favor of Beşiktaş is approximately 0.4%. Other results are seen clearly in the table.

## V. CONCLUSION

In the present study, the number of goals scored between the time intervals [0-15], [16-30], [31-45], [46-60], [61-75] and [76-90] in the football matches by the 4 big teams in the Turkish Super League throughout the last 8 years were determined. Accordingly, the probabilities of scoring goals were calculated using probability functions and parameters with the nonhomogeneous Poisson process. Galatasaray scored between 72-127, Fenerbahçe between 62-127, Beşiktaş between 62-130 and Trabzonspor between 58-103 in these time intervals over the last 8 years. The goal minutes when the most goals scored are found to be between the minutes 76-90. The probability of the teams scoring goals between the minutes 76-90 which is when the most goals are scored is found as Beşiktaş > Galatasaray = Fenerbahçe > Trabzonspor.

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