

Optimal Control for an Industrial Robot Manipulator

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Abstract: This paper presents the optimal control problem of a nonlinear industrial robot manipulator in the absence of holonomic constraint force based on the point of view of adaptive dynamic programming. The Adaptive Dynamic Programming algorithm employs techniques to tune the actor-critic network simultaneously to approximate the control policy and cost function. The convergence of weight, as well as position tracking control problem, was considered by theoretical analysis.

Keywords: Industrial Robot Manipulator, Input/output Constraint, Optimal control.

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I. INTRODUCTION

The control methodology for robotic manipulators has been widely developed [1-15]. The main challenges of the control design have been considered, such as robust adaptive control problems, motion/force control, input saturation, and entire state constraints [16-23]. Several control techniques have been employed for manipulators to tackle the issue of input saturation by adding more terms into the designed control input considering the absence of input constraint [6,11,27]. In [6,8], the authors proposed a new control system reference due to the input saturation. The additional term world is computed based on the derivative of the previous Lyapunov candidate function along the state trajectory under the control input saturation.

Furthermore, authors in [11-13] give a new approach to addressing the input constraints and handling the disturbances. The proposed sliding surface was employed the Sat function of joint variables. The equivalent sliding mode control algorithm was designed then the boundedness of control input was estimated. The advantage of this approach is that input boundedness is absolutely adjusted by selecting several parameters. The authors in [27-29] present a technique to implement the input constraint using a modified Lyapunov Candidate function. Because of the actuator saturation, the Lyapunov function would be more the quadratic term from the difference between the control input from the controller and the real signal applied to the object. The control design was obtained after considering the Lyapunov function derivative along the system trajectory. Optimal control algorithm obtains the control design that can tackle the input, state constraint based on considering the optimization problem in the presence of constraint

II. CONTROL DESIGN

Consider the following robot manipulator without constraint:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d(t) = \tau \quad (1)$$

Assumption 1. The inertia matrix $M(q)$ is symmetric, positive definite, and guarantees the inequality $\forall \xi(t) \in \mathbb{R}^n$ as follows:

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq \bar{m}(q) \|\xi\|^2 \quad (2)$$

where $m_1 \in \mathbb{R}^+$, $\bar{m}(q) \in \mathbb{R}^+$, $\|\cdot\|$ is a known positive constant, a known positive function, and the standard Euclidean norm.

Assumption 2. The relationship between an inertia matrix $M(q)$ and the Coriolis matrix $C(q, \dot{q})$ can be represented as follows:

$$\xi^T (\dot{M}(q) - 2C(q, \dot{q}))\xi = 0 \quad \forall \xi \in \mathbb{R}^n \quad (3)$$

It should be noticed that this manipulator is considered in the absence of Holonomic constraint force.

By using the control input (4) for manipulator (1) with nonlinear function (5) obtaining from (6), (7), (8), we lead to the nonlinear model (9):

$$u = -\tau + h + \tau_d \quad (4)$$

$$h = M (\alpha_1 \dot{e}_1) + C (\alpha_1 e_1) + G (q) + F (\dot{q}) \quad (5)$$

$$e_1 = q_d - q \quad (6)$$

$$e_2 = \dot{e}_1 + \alpha_1 e_1 \quad (7)$$

$$r = \dot{e}_2 + \alpha_2 e_2 \quad (8)$$

$$\dot{x} = f(x) + g(x)u \quad (9)$$

Where:

$$x = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, f(x) = \begin{bmatrix} -\alpha_1 & I_{n \times n} \\ 0_{n \times n} & -M^{-1}C \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, g(x) = \begin{bmatrix} 0_{n \times n} \\ -M^{-1} \end{bmatrix}.$$

Now, the control object is to design a control law u to guarantee not only stabilization (9) but also minimizing the quadratic cost function with infinite horizon as follows:

$$V(x_0) = \int_0^{\infty} r(x, u) dt \quad (10)$$

$$r(x, u) = Q(x) + u^T R u \quad (11)$$

In which, $Q(x)$ and R is a positive definite function of x , symmetric definite positive matrix, respectively.

Consider the following affine system

$$\dot{x} = f(x) + g(x)u \quad (12)$$

where $x \in \mathcal{X} \subseteq R^n$, $u \in U \subseteq R^m$. $f(x)$ and $g(x)$ satisfy Lipschitz condition and $f(0) = 0$.

The cost function is defined as (10).

A control policy $\mu(x)$ is defined as an admissible policy if $\mu(x)$ stabilize system (12), and the equivalent value function $V^\mu(x)$ is finite. $\Psi(\mathcal{X})$ is denoted set of admissible control policy.

For any admissible policy $\mu(x)$, the nonlinear Lyapunov Equation can be formulated

$$r(x, \mu(x)) + \left(\frac{\partial V}{\partial x} \right)^T (f(x) + g(x)\mu(x)) = 0 \quad (13)$$

Defining Hamilton function and optimal cost function as follows:

$$H(x, \mu, V_x) = r(x, \mu) + (V_x^\mu)^T (f(x) + g(x)\mu) \quad (14)$$

$$V^*(x) = \min_{\mu \in \Psi(\mathcal{X})} \left(\int_t^{\infty} r(x, \mu) \right)$$

We lead to the following Lyapunov Equation equation:

$$0 = \min_{\mu \in \Psi(\mathcal{X})} H(x, \mu, V_x^*) = H(x, \mu^*, V_x^*) \quad (15)$$

It can be noticed that, μ^* is optimal policy corresponding with the optimal cost function and $H(x, \mu, V_x^\mu) = 0$

Now, the optimal control policy can be obtained by taking the derivative of Hamilton problem with respect to policy μ

$$\mu^* = -\frac{1}{2} (R^{-1} g^T V_x^*) \quad (16)$$

This work present Policy Iteration algorithm for a robot manipulator including 2 steps as follows:

Initiate admissible control policy $\mu^0(x)$

Repeat

- Step 1: Policy Evaluation
Solve NLE for $V^i(x)$ corresponding given control policy μ^i

$$r(x, \mu^i(x)) + (V_x^i)^T (f(x) + g(x) \mu^i(x)) = 0 \quad (17)$$

- Step 2: Policy improvement
Update new policy according to

$$\mu^{i+1} = -\frac{1}{2} (R^{-1} g^T V_x^i) \quad (18)$$

Until $n = n_{\max}$ **or** $|V^{i+1} - V^i| \leq \varepsilon_v$.

Where n_{\max} is a number of limited iteration and ε_v is arbitrary given a small positive numbers

This algorithm proves each policy control μ^i is admissible control. The cost function V^i was reduced at each step until it converged to optimal policy and μ^i converged toward optimal policy.

III. CONCLUSIONS

This paper mentioned the problem of optimal control design for a. With the optimal technique, the solution algorithm to obtain the controller satisfies the convergence of weight and the position tracking. The algorithm's intersection with the general object has been demonstrated in this study. The author will apply a specific object and the system simulation in the next research in the future.

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REFERENCES

- [1]. Dupree, K., Patre, P. M., Wilcox, Z. D., & Dixon, W. E. (2011). Asymptotic optimal control of uncertain nonlinear Euler–Lagrange systems. *Automatica*, 47(1), 99-107
- [2]. Zhang, J. X., & Yang, G. H. (2020). Fault-tolerant output-constrained control of unknown Euler–Lagrange systems with prescribed tracking accuracy. *Automatica*, 111, 108606.
- [3]. Meng, Z., & Huang, P. (2014). An effective approach control scheme for the tethered space robot system. *International Journal of Advanced Robotic Systems*, 11(9), 140.
- [4]. Pukdeboon, C. (2013). Lyapunov optimizing sliding mode control for robot manipulators. *Applied Mathematical Sciences*, 7(63), 3123-3139.
- [5]. Wu, H., & Xu, D. (2019, July). Inverse optimality and adaptive asymptotic tracking control of uncertain Euler-Lagrange systems. In *2019 IEEE 15th International Conference on Control and Automation (ICCA)* (pp. 242-247). IEEE
- [6]. Hu, X., Wei, X., Zhang, H., Han, J., & Liu, X. (2019). Robust adaptive tracking control for a class of mechanical systems with unknown disturbances under actuator saturation. *International Journal of Robust and Nonlinear Control*, 29(6), 1893-1908.
- [7]. Zhou, Q., Chen, G., Lu, R., & Bai, W. (2019). Disturbance-observer-based event-triggered control for multi-agent systems with input saturation. *Scientia Sinica Informationis*, 49(11), 1502-1516.
- [8]. Hu, X., Wei, X., Han, J., & Zhu, X. (2020). Adaptive disturbance estimation and cancelation for ships under thruster saturation. *International Journal of Robust and Nonlinear Control*, 30(13), 5004-5020.
- [9]. Bai, Y., Cao, Y., & Li, T. (2019). Optimized backstepping design for ship course following control based on actor-critic architecture with input saturation. *IEEE Access*, 7, 73516-73528.
- [10]. Hu, X., Wei, X., Kao, Y., & Han, J. (2021). Robust synchronization for under-actuated vessels based on disturbance observer. *IEEE Transactions on Intelligent Transportation Systems*.
- [11]. Guo, Y., Huang, B., Li, A. J., & Wang, C. Q. (2019). Integral sliding mode control for Euler- Lagrange systems with input saturation. *International Journal of Robust and Nonlinear Control*, 29(4), 1088-1100.
- [12]. Sun, T., Zhang, X., Yang, H., & Pan, Y. (2022). Singular perturbation-based saturated adaptive control for underactuated Euler–Lagrange systems. *ISA transactions*, 119, 74-80.
- [13]. Shao, K., Tang, R., Xu, F., Wang, X., & Zheng, J. (2021). Adaptive sliding mode control for uncertain Euler–Lagrange systems with input saturation. *Journal of the Franklin Institute*, 358(16), 8356-8376.
- [14]. Umoh, E. A., & Tola, O. J. (2021). Robust Global Synchronization of a Hyperchaotic System with Wide Parameter Space via Integral Sliding Mode Control Technique. *International Journal of Robotics and Control Systems*, 1(4), 453-462.
- [15]. Cao, G., Shi, H., Zhang, X., Zhu, Z., Liu, Y., & Yu, H. (2021, October). Continuous Super-Twisting Observer-Based Super-Twisting Control of Euler-Lagrange Systems. In *International Conference on Intelligent Robotics and Applications* (pp. 455-465). Springer, Cham.
- [16]. He, W., Chen, Y., & Yin, Z. (2015). Adaptive neural network control of an uncertain robot with full-state constraints. *IEEE transactions on cybernetics*, 46(3), 620-629.

- [17]. Liu, Y. J., & Tong, S. (2017). Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems. *Automatica*, 76, 143-152.
- [18]. Yang, C., Jiang, Y., Li, Z., He, W., & Su, C. Y. (2016). Neural control of bimanual robots with guaranteed global stability and motion precision. *IEEE Transactions on Industrial Informatics*, 13(3), 1162-1171.
- [19]. He, W., & Dong, Y. (2017). Adaptive fuzzy neural network control for a constrained robot using impedance learning. *IEEE transactions on neural networks and learning systems*, 29(4), 1174-1186.
- [20]. Yin, X., & Pan, L. (2018). Enhancing trajectory tracking accuracy for industrial robot with robust adaptive control. *Robotics and Computer-Integrated Manufacturing*, 51, 97-102.
- [21]. Liu, Y. J., Zeng, Q., Tong, S., Chen, C. P., & Liu, L. (2019). Adaptive neural network control for active suspension systems with time-varying vertical displacement and speed constraints. *IEEE Transactions on Industrial Electronics*, 66(12), 9458-9466.
- [22]. Kong, L., He, W., Yang, C., Li, Z., & Sun, C. (2019). Adaptive fuzzy control for coordinated multiple robots with constraint using impedance learning. *IEEE transactions on cybernetics*, 49(8), 3052-3063.
- [23]. Li, D. P., & Li, D. J. (2017). Adaptive neural tracking control for an uncertain state constrained robotic manipulator with unknown time-varying delays. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 48(12), 2219-2228.
- [24]. He, S., Dai, S. L., & Luo, F. (2018). Asymptotic trajectory tracking control with guaranteed transient behavior for MSV with uncertain dynamics and external disturbances. *IEEE Transactions on Industrial Electronics*, 66(5), 3712-3720.
- [25]. Yang, Z., Peng, J., & Liu, Y. (2019). Adaptive neural network force tracking impedance control for uncertain robotic manipulator based on nonlinear velocity observer. *Neurocomputing*, 331, 263-280.
- [26]. Zhang, S., Yang, P., Kong, L., Chen, W., Fu, Q., & Peng, K. (2019). Neural networks-based fault tolerant control of a robot via fast terminal sliding mode. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*.
- [27]. He, W., Dong, Y., & Sun, C. (2015). Adaptive neural impedance control of a robotic manipulator with input saturation. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 46(3), 334-344.
- [28]. Liu, Y. J., & Tong, S. (2017). Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems. *Automatica*, 76, 143-152.
- [29]. A. Garg, S. Kumar and K. Dutta, "An analytical survey of state of the art Jellyfish attack detection and prevention techniques", 2016 Fourth International Conference on Parallel, Distributed and Grid Computing (PDGC), 2016.