Research Inventy: International Journal of Engineering And Science
Vol.7, Issue 4 (April 2017), PP -11-16
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Elastic Metamaterials Analysis: Simple and Double Resonators

[1]G. S. Rodrigues, [2]H. I. Weber
[1](UNESA, Estácio de Sá University, Brazil)
[2](PUC-Rio,Pontifical Catholic University of Rio de Janeiro, Brazil)

ABSTRACT– *Metamaterials are broadly defined as a material with properties not found in nature,i.e, a typically man made material especially designed to achieve certain unusual properties. First applications of metamaterials were in field of electromagnetic waves and some features of these materials were negative permeativity and negative refraction index. Due to similarity with electromagnetic waves, mechanical waves propagation through metamaterials began to be studied and the concept of elastic metamaterials was created. The main property of elastic metamaterial is the negative dynamic effective mass where the wave propagation is blocked.It will be discussed two kinds of unit cell that compound the elastic metamaterial and how some changes in parameters affect the metamaterial negative effective mass.*
Keywords –*Double Resonator, Metamaterial, Negative Effective Mass, Single Resonator.*

I. INTRODUCTION

The use of metamaterials applied in mechanical waves mitigation is a relatively recent area of science. By definition, these materials are specially constructed and they present features that are not found in materials in nature. The interesting characteristic the elastic metamaterials used to mitigate mechanical waves is the negative effective mass density. This property is obtained by the insertion of masses linked by springs that work as internal resonators and it is observed that these metamaterials act as a filter when the mechanical waves propagate in the structure with frequency of propagation near the resonant frequency of the internal resonator. In this way, the metamaterials prevent the propagation of the wave or reduce its intensity. The range of frequency where the metamaterial prevent the propagation of the wave is called bandgap.

The applications of metamaterials were first used in the field of electromagnetic waves and due to the mathematical analogy between the acoustic and electromagnetic waves, the acoustic metamaterials recently began to be explored, being created the elastic metamaterial concept.

Some studies [1, 2], for example, showed the use of metamaterials to mitigate shock waves and a significant reduction of pulse intensity was observed.

In this paper, we present the fundamental theory of the unit cell of elastic metamaterial and its two basic elements: the single and the double resonators.

II. SINGLE RESONATOR

As mentioned, applications using metamaterials were first used in the field of electromagnetic waves, where researchers investigated negative electrical permittivity, negative magnetic permeability [3] and negative refractive index [4]. Due to the mathematical analogy of acoustic and electromagnetic waves, counteracting acoustic metamaterials were recently explored. This new branch of acoustic metamaterials, also known as mechanical or elastic metamaterials, consists of adapted microstructures that present unusual mechanical properties, such as negative effective modulus of elasticity and / or negative effective mass density. A system containing negative mass properties consists of a chain of mass units, each mass being bound by a spring in an internal mass.

This resonator unit is shown in Fig. 1, where the outer cell unit has mass $m_1$and displacement u_1. The internal resonator has mass m_2 and displacement u_2. The stiffness of the spring is linear with stiffness coefficient k_2 and connects external mass to the internal resonator.

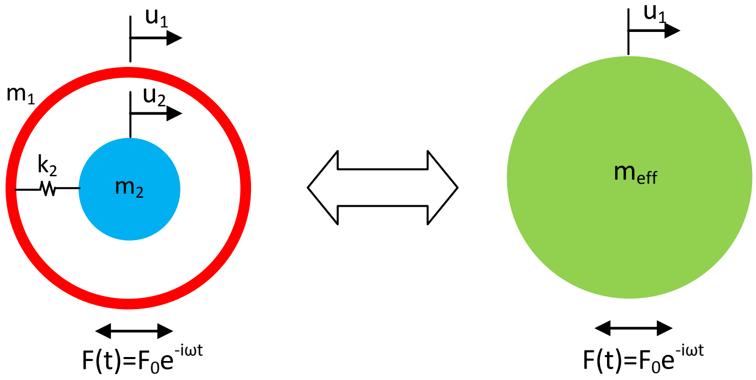

Figure 1 – Single resonator and equivalent effective mass

The free-body diagram of each of the masses m_1 and m_2 provides the following equations:

$$m_1\ddot{u}_1=F+k_2(u_2-u_1) \quad 1$$

$$m_2\ddot{u}_2=-k_2(u_2-u_1) \quad 2$$

Considering the mass displacements as a harmonic wave behavior, as well as the applied force, according to (3) and (4):

$$u_\gamma(x,t)=\hat{u}_\gamma e^{-i\omega t} \quad 3$$

$$F(t)=F_0 e^{-i\omega t} \quad 4$$

Where γ = 1, 2, corresponding to the masses m_1 and m_2 . (1) and (2) can be solved by being replaced in(3) and (4). Thus, the simplified relation is presented in (5):

$$0=F_0+\left(m_1+\frac{\omega_2^2 m_2}{\omega_2^2-\omega^2}\right)\omega^2\hat{u}_1 \quad 5$$

Where $\omega_2=\sqrt{k_2/m_2}$ is the natural frequency of the internal mass resonator m_2. Applying the equation of motion in the effective mass m_{eff} based on (4), we obtain the relation:

$$F_0=-m_{eff}\omega^2\hat{u}_1 \quad 6$$

From (5) and (6), we conclude that the effective mass m_{eff} is given by:

$$m_{eff}=\left(m_1+\frac{\omega_2^2 m_2}{\omega_2^2-\omega^2}\right) \quad 7$$

It is observed in (7) that the effective mass, m_{eff}, is dependent on the frequency ω. Assuming m_{st} as the static mass (m_1+m_2), the normalized effective mass can be obtained m_{eff}/m_{st} :

$$\frac{m_{ef}}{m_{st}} = 1 + \frac{\theta}{1+\theta}\left[\frac{\left(\omega/\omega_2\right)^2}{1-(\omega/\omega_2)^2}\right] \qquad 8$$

Where θ is the ratio of the internal mass, m_2, and the external mass, m_1. Figure 2 shows how the ratio m_{eff}/m_{st} varies when ω/ω_2 is considered in the abscissa axis. It can be seen that there is a narrow band gap band near the local resonance frequency of the internal mass m_2. Huang et. al. (2009) [5] showed that this negative effective mass corresponds to the band gap region of the dispersion curve when the wave propagation is considered.

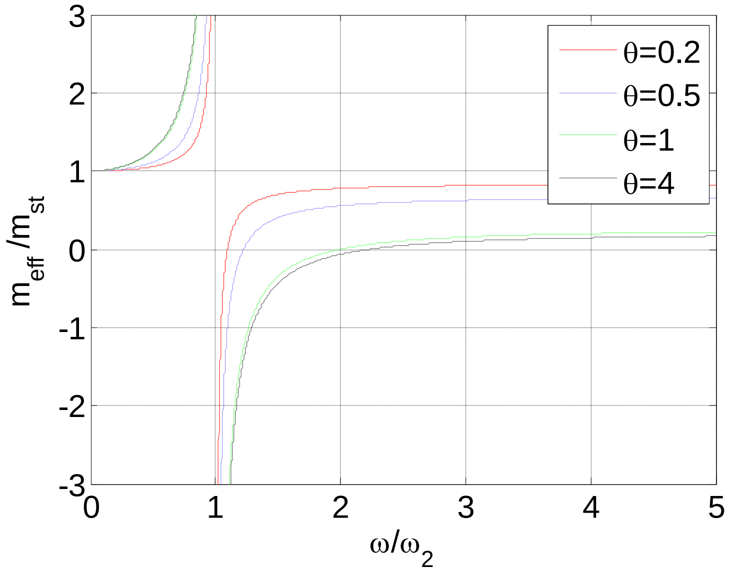

Figura2 – Normalized effective mass versus ω/ω_2 ratio (single resonator)

III. Double Resonator

As depicted in section II, the unit cell formed by a single resonator represents a system with one degree of freedom. In this section, it will be presented a unit cell formed by a double resonator forming a system with two degree of freedom.

Figure 3 shows the unit cell proposed by Tan et. al. (2012) [6] representing a spring mass system with a double resonator on the left and its corresponding individual effective mass on the right.

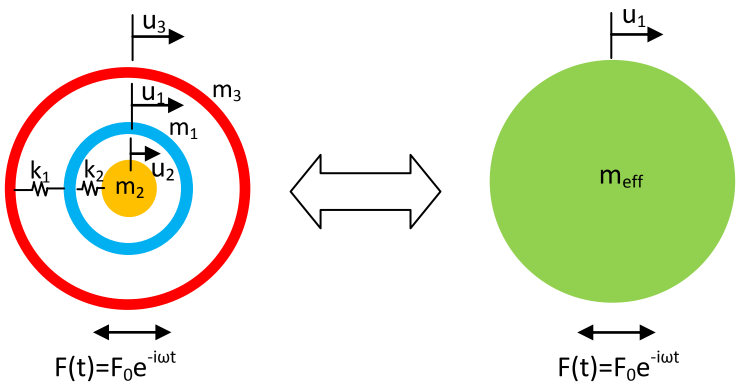

Figure 3 – Double resonator and equivalent effective mass

In this case, the normalized effective mass, m_{eff}/m_{st}, can be obtained by assuming the static mass $m_{st} = (m_1 + m_2 + m_3)$ and the factor $\theta_{st} = \theta_1 + \theta_3 + \theta_1\theta_3$, as:

$$\frac{m_{eff}}{m_{st}} = \frac{\theta_1}{\theta_{st}} + \frac{\theta_3}{\theta_{st}}\left[\frac{1+\theta_1-(\omega/\omega_2)^2}{[1-(\omega/\omega_2)^2][(1+\delta_1-(\delta_1/\theta_1))(\omega/\omega_2)^2]-\delta_1}\right] \qquad 9$$

Where $\theta_1 = m_2/m_1$, $\theta_3 = m_2/m_3$ and $\delta_1 = k_2/k_1$. The normalized effective mass versus the ratio ω/ω_2 is shown in Fig.4 for fixed values of θ_1 and θ_3 and three different values for δ_1.

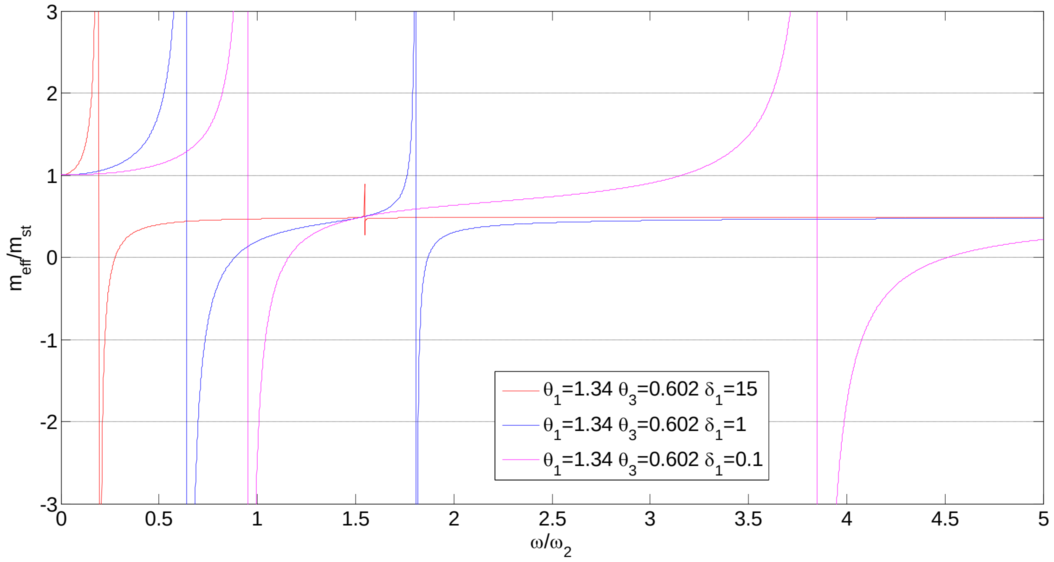

Figure 4 – Normalized effective mass versus ω/ω_2 ratio. θ_1 and θ_3 fixed. (double resonator)

In Fig. 5, one can observe the normalized effective mass versus the ratio ω/ω_2 for θ_1 and δ_1 fixed and varying θ_3.

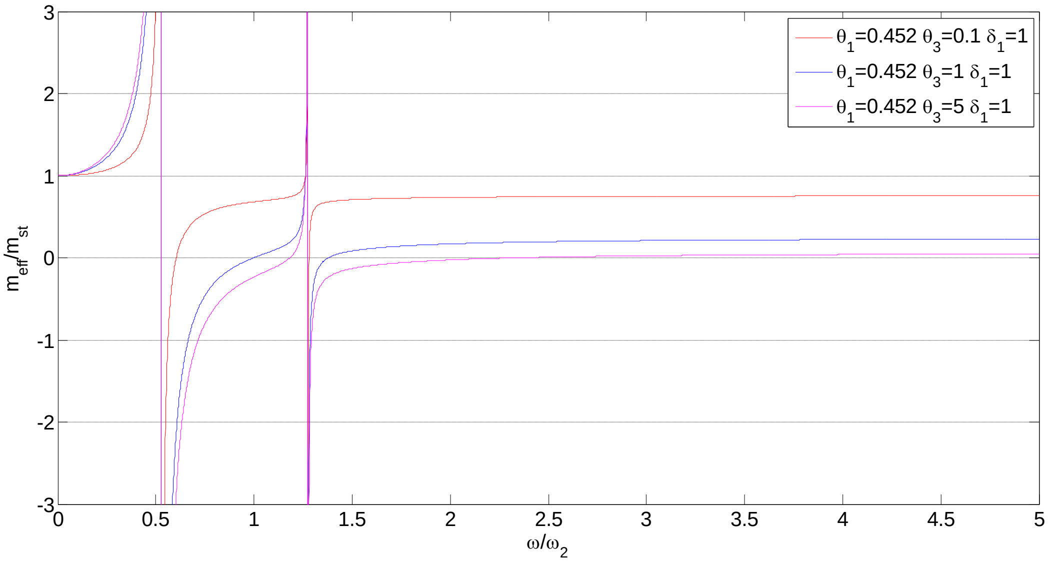

Figure 5 – Normalized effective mass versus ω/ω_2 ratio. θ_1 and δ_1 fixed. (double resonator)

In Fig. 6, it is also plotted the normalized effective mass versus the ratio ω/ω_2 for θ_1 and δ_1 fixed and varying θ_3, but for a bigger value of δ_1.

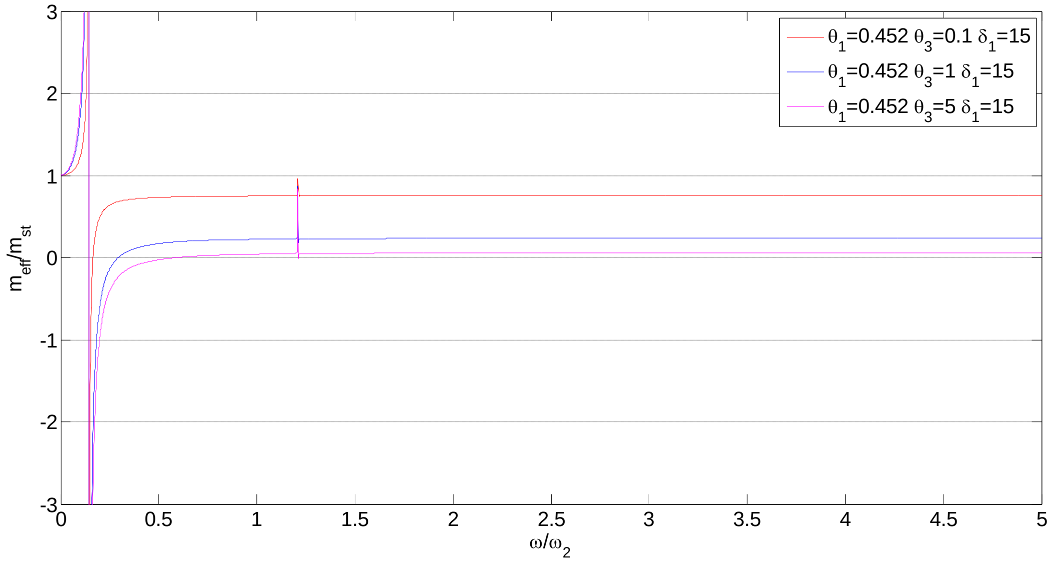

Figure 6 – Normalized effective mass versus ω/ω_2 ratio. θ_1 and δ_1 fixed. (double resonator)

When we fix θ_3 and δ_1 and use different values for θ_1, we obtain a graph similar to the one shown in Fig. 7.

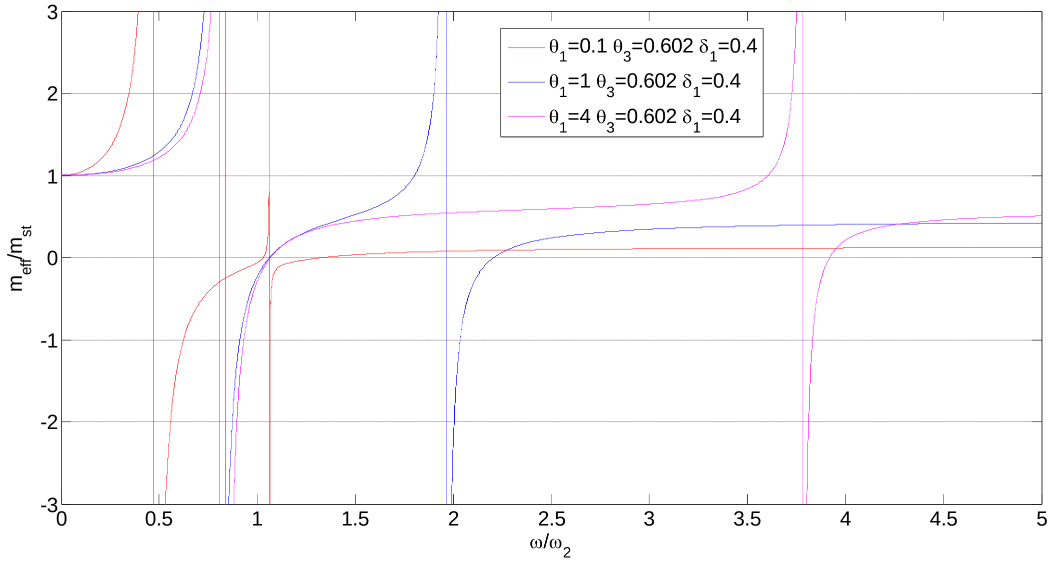

Figure 7 – Normalized effective mass versus ω/ω_2 ratio. θ_3 and δ_1 fixed. (double resonator)

IV. RESULTS AND DISCUSSION

Single resonator analysis shows that we are not able to modify substantially the system varying the parameters once the only variable is mass ratio θ. For small values of this ratio, we observe a narrow range of negative effective mass. As this ratio increases, the range of negative effective mass also increase, i.e., as internal mass m_2 increases comparatively to mass m_1 the negative effective mass range becomes wider. Obviously there is a constructive limit of the metamaterial not considered here. As can be seen in Fig. 2, for $\theta = 1$, we have a negative effective mass between $1 < \omega/\omega_2 < 2$. That means there is a wave propagation bandgap for frequencies between the natural frequency and twice its value. We also observed the band gap doesn't change substantially for $\theta = 4$, in other words, despite we have mass m_2 four times bigger than mass m_1,the range of stopping wave propagation is almost the same.

When considering double resonators we are able to manipulate more parameters that influence the region of negative effective mass, namely $\theta_1 = m_2/m_1$, $\theta_3 = m_2/m_3$ and $\delta_1 = k_2/k_1$. For θ_1 and θ_3 fixed and large values of δ_1, Fig. 4 (red line) shows that only one region of negative effective mass appears. For smaller values of δ_1, we obtain a wider range of bandgap in two distinct regions of frequency. When we fix δ_1 and θ_1 and use different values of θ_3, the regions of frequency keep the same but as θ_3 increases the negative effective mass becomes wider. It must be said that this occurs for small values of δ_1. Developing the same simulation for δ_1 and θ_1 fixed but $\delta_1 \gg 1$, we obtain a result similar to only one resonator as presented in Fig. 6. The last consideration is when δ_1 and θ_3 are fixed and we vary θ_1 as shown in Fig. 7. Here we note a wide range of negative effective mass in the first resonance frequency for small values of θ_1 and a narrow range of negative effective mass for the second resonance frequency for small values of θ_1. For medium and large values of θ_1, we observe a medium range of negative effective mass but for bigger values of θ_1, the second bandgap occurs in higher frequencies.

V. CONCLUSION

Clearly we observe that double resonators are more efficient than single resonators. There are much more options to modify in design to exactly achieve the range of frequency in which occurs the negative effective mass phenomena. However, comparing the single and double resonator metamaterials, the first one is simpler and obviously it is an advantage in terms of manufacturing. By being more complex and encompass more constructive parts, double resonator metamaterial is more complicated to develop three-dimensionally and so achieve the design parameters in practice.

ACKNOWLEDGEMENTS

This work was supported by the Productivity Research Program of the Estácio de Sá University.

REFERENCES

[1] K. T, Tan, C. T. Sun, Metacomposites Protection System against Primary Blast Injury. *Proceedings of the 29thAmerican Society for Composites*, La Jolla, CA, 2014.

[2] J. M. Manimala, H. H. Huang, C. T. Sun, R. Snyder, S. Bland, Dynamic load mitigation using negative effective mass structures. *Engineering Structures, (80)*, 2014, 458-468.

[3] V. G.Veselago, The electrodynamics of substances with simultaneously negative values of ε and μ. *Soviet physics uspekhi, 10(4)*,1968, 509.

[4] J. B.Pendry, Negative refraction makes a perfect lens. *Physical review letters, 85 (18)*, 2000, 3966.

[5] H. H. Huang, C. T. Sun,G. L.Huang, On the negative effective mass density in acoustic metamaterials. *International Journal of Engineering Science, 47(4)*, 2009, 610-617.

[6] K. T. Tan, H. H. Huang, C. T. Sun, Optimizing the band gap of effective mass negativity in acoustic metamaterials. *Applied Physics Letters, 101(24)*, 2012, 241902.