

## Nonlinear Adaptive Control for Welding Mobile Manipulator

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**ABSTRACT** : This paper presents an adaptive tracking control method for a welding mobile manipulator with a kinematic model in which several unknown dimensional parameters exist. The mobile manipulator consists of the manipulator mounts on a mobile-platform. Based on the Lyapunov function, controllers are designed to guarantee stability of the whole system when the end-effector of the manipulator performs a welding task. The update laws are also designed to estimate the unknown dimensional parameters. The simulation and experimental results are presented to show the effectiveness of the proposed controllers.

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### I. INTRODUCTION

Nowadays, the welding robots become widely used in the tasks which are harmful and dangerous for the workers. The three-linked planar manipulator with a torch mounted at the end-effector has three degree-of-freedom (DOF). Hence it should be satisfies for three constraints of welding task such as two direction constraints and one heading angle constraint. However, because a fixed manipulator has a small work space, a mobile manipulator is used for increasing the work space by placing the manipulator on the two-wheeled mobile-platform. The mobile manipulator has five DOF such as three DOF of manipulator and two DOF of mobile-platform, whereas the welding task requires only three DOF. Hence, the mobile manipulator is a kinematic redundant system. To solve this problem, two constraints of linear and rotational velocity constraints of mobile-platform are combined into the mobile manipulator system. This means that the mobile-platform motion is not a free motion but it is a constrained motion. The target of two constraints is to move the configuration of manipulator into its initial configuration to avoid the singularity.

In recent years, the analysis and control of the robots are studied by many researchers. Most of them developed the control algorithm using the dynamic model or combining the dynamic model with the kinematic model. Fierro and Lewis [5] developed a combined kinematic/torque control law of a wheeled mobile robot by using a back-stepping approach and asymptotic stability is guaranteed by Lyapunov theory. Yamamoto and Yun [6] developed a control algorithm for the mobile manipulator so that the manipulator is always positioned at the preferred configurations measured by its manipulability to avoid the singularity. Seraji [2] developed a simple on-line coordinated control of mobile manipulator, and the redundancy problem is solved by introducing a set of user-specified additional task during the end-effector motion. He defined a scalar cost function and minimized it to have non-singularity. Yoo, Kim and Na [3] developed a control algorithm for the mobile manipulator based on the Lagrange's equations of motion. Jeon, Kam, Park and Kim [7] applied the two-wheeled mobile robot with a torch slider welding automation. They proposed a seam-tracking and motion control of the welding mobile robot for lattice-type welding. Fukao, Nakagawa and Adachi [4] developed an adaptive tracking controller for the kinematic/dynamic model with the existing of unknown parameters, but the algorithm in their paper is applied only to the mobile robots. Phan, et al [8] proposed a decentralized control method for five DOF mobile – manipulator with exactly known with parameters. These methods reveal that the adaptive control method for the kinematic model of the mobile manipulator has studied insufficiently. For this reason, an adaptive control algorithm for the kinematic model of the mobile manipulator is the study target of this paper. It is assumed that all dimensional parameters of the mobile manipulator are unknown and the controllers estimate them by using the update laws. Finally, the simulation and the experimental results are presented to show the effectiveness of the proposed method.

### II. SYSTEM MODELING

#### A Kinematic Equations of The Manipulator

Consider a three-linked manipulator as shown in Fig. 1. A Cartesian coordinate frame is attached at the joint 1 of the manipulator. Because this frame is fixed at the center point of mobile-platform and moves in the world frame (Frame  $X\_Y$ ), this frame is called the moving frame (Frame  $x\_y$ ).

The velocity vector of the end-effector with respect to the moving frame is given by

$${}^1V_E = J\dot{\theta}. \quad (1)$$

where  ${}^1V_E = [\dot{x}_E \ \dot{y}_E \ \dot{\phi}_E]^T$  is the velocity vector of the end-effector with respect to the moving frame,  $\dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T$  is the angular velocity vector of the joints of the manipulator, and  $J$  is the Jacobian matrix.

$$J = \begin{bmatrix} -L_3S_{123} - L_2S_{12} - L_1S_1 & -L_3S_{123} - L_2S_{12} & -L_3S_{123} \\ L_3C_{123} + L_2C_{12} + L_1C_1 & L_3C_{123} + L_2C_{12} & L_3C_{123} \\ 1 & 1 & 1 \end{bmatrix} \quad (2)$$

where  $L_1, L_2, L_3$  are the length of links of the manipulator,  $S_1 = \sin(\theta_1)$ ,  $S_{12} = \sin(\theta_1 + \theta_2)$ ,  $S_{123} = \sin(\theta_1 + \theta_2 + \theta_{31})$ ,  $C_1 = \cos(\theta_1)$ ,  $C_{12} = \cos(\theta_1 + \theta_2)$ ,  $C_{123} = \cos(\theta_1 + \theta_2 + \theta_{31})$ .

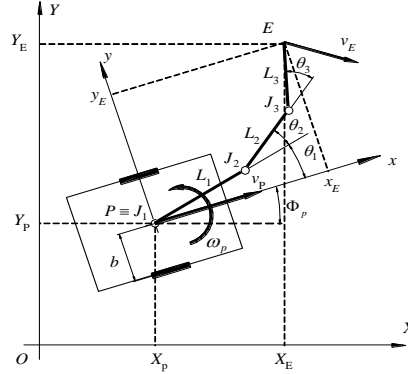


Fig. 1

Scheme for deriving the kinematic equations of the mobile manipulator

The end-effector's velocity vector with respect to the world frame from the motion equation of a rigid body in a plane is obtained as follows

$$V_E = V_P + W_P \times {}^0Rot_1 {}^1p_E + {}^0Rot_1 V_E \quad (3)$$

where:

$V_E$ : the velocity vector of the end-effector with respect to the world frame

$V_P$ : the velocity vector of the center point of platform with respect to the world frame

$W_P$ : the rotational velocity vector of the moving frame

${}^0Rot_1$ : the rotation transform matrix from the moving frame to the world frame

${}^1p_E$ : the position vector of the end-effector with respect to the moving frame

$$V_E = \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{\Phi}_E \end{bmatrix}, \quad V_P = \begin{bmatrix} \dot{X}_P \\ \dot{Y}_P \\ \dot{\Phi}_P \end{bmatrix}, \quad W_P = \begin{bmatrix} 0 \\ 0 \\ \dot{\Phi}_P \end{bmatrix}, \quad {}^1p_E = \begin{bmatrix} x_E \\ y_E \\ \phi_E \end{bmatrix},$$

$${}^0Rot_1 = \begin{bmatrix} \cos \Phi_P & -\sin \Phi_P & 0 \\ \sin \Phi_P & \cos \Phi_P & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\Phi_E = \theta_1 + \theta_2 + \theta_3 + \Phi_P - \frac{\pi}{2}, \quad \dot{\Phi}_E = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 + \dot{\Phi}_P$$

## B Kinematic Equations of the Mobile Platform

When the mobile-platform moves in a horizontal plane, it obtains the linear velocity  $v_p$  and the angular velocity  $\omega_p$ . The relation between  $v_p$ ;  $\omega_p$  and the angular velocities of two driving wheels is given by

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} 1/r & b/r \\ 1/r & -b/r \end{bmatrix} \begin{bmatrix} v_p \\ \omega_p \end{bmatrix}. \quad (4)$$

where  $\omega_{rw}, \omega_{lw}$  are the angular velocities of the right and left wheels.

### III. CONTROLLERS DESIGN

#### A. Controller Design For Manipulator

It is assumed that the dimensional parameters of  $L_1$ ,  $L_2$  and  $L_3$  are known exactly. The coordinate of the mobile manipulator with the reference welding trajectory is shown in Fig. 2. Our objective is to design a controller so that the end-effector with the coordinates  $E(X_E, Y_E, \Phi_E)$  tracks to the reference point  $R(X_R, Y_R, \Phi_R)$ . The tracking error vector  $E_E = [e_1 \ e_2 \ e_3]^T$  is defined as follows

$$E_E = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\Phi_E & \sin\Phi_E & 0 \\ -\sin\Phi_E & \cos\Phi_E & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_R - X_E \\ Y_R - Y_E \\ \Phi_R - \Phi_E \end{bmatrix}. \quad (5)$$

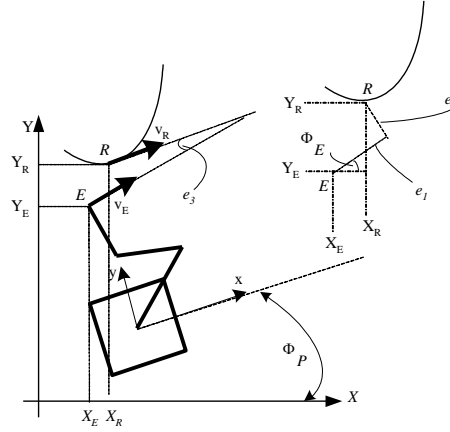


Fig. 2

Scheme for deriving the error equations of the manipulator

Let us denote

$$A = \begin{bmatrix} \cos\Phi_E & \sin\Phi_E & 0 \\ -\sin\Phi_E & \cos\Phi_E & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad p_R = \begin{bmatrix} X_R \\ Y_R \\ \Phi_R \end{bmatrix}, \quad p_E = \begin{bmatrix} X_E \\ Y_E \\ \Phi_E \end{bmatrix}$$

Equation (5) can be re-expressed as follows

$$E_E = A(p_R - p_E). \quad (6)$$

and its derivative is

$$\dot{E}_E = \dot{A}(p_R - p_E) + A(\dot{p}_R - \dot{p}_E) \quad (7)$$

$$\dot{E}_E = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega_E e_2 - v_E + v_R \cos e_3 \\ -\omega_E e_1 + v_R \sin e_3 \\ \omega_R - \omega_E \end{bmatrix}$$

where  $\dot{p}_R = V_R$  and  $\dot{p}_E = V_E$ . Substituting (1), (3) and (6) into (7) yields

$$\dot{E}_E = \dot{A}A^{-1}E_E + A(V_R - (V_P + W_P \times^0 \text{Rot}_1^1 p_E + {}^0 \text{Rot}_1 J \dot{\theta})) \quad (8)$$

The chosen Lyapunov function and its derivative are

$$V_0 = \frac{1}{2} E_E^T E_E \quad (9)$$

$$\dot{V}_0 = E_E^T \dot{E}_E \quad (10)$$

To achieve the negative of  $\dot{V}_0$ , (11) must satisfy

$$\dot{E}_E = -KE_E \quad (11)$$

where  $K = \text{diag}(k_1 \ k_2 \ k_3)$  with positive values  $k_1, k_2$  and  $k_3$ .

Substituting (11) into (8) yields

$$KE_E = -\dot{A}A^{-1}E_E - A(V_R - (V_P + W_P \times^0 \text{Rot}_1^{-1} p_E + {}^0 \text{Rot}_1 J \dot{\theta})) \quad (12)$$

If  $V_P$  is chosen in advanced as a constraint, the errors converge to zero if the non-adaptive control law of the manipulator is given as follows

$$\dot{\theta} = J^{-1} {}^0 \text{Rot}_1^{-1} (A^{-1} (\dot{A}A^{-1} + K)E_E + V_R - V_P - W_P \times^0 \text{Rot}_1^{-1} p_E) \quad (13)$$

(13) is the controller of the manipulator, and it can be re-expressed as follows

$$\begin{aligned} \dot{\theta}_1 &= \frac{1}{L_1 S_2} \left( v_R S_{3e_3} - v_P C_{12} + (e_2 k_2 - e_1 \omega_E) C_3 \right. \\ &\quad \left. + (e_1 k_1 + e_2 \omega_E + L_3 (\omega_R + e_3 (k_3 + \omega_E))) S_3 \right) - \omega_P \\ \dot{\theta}_2 &= \frac{1}{L_1 L_2 S_2} \left( -v_R (L_1 S_{23e_3} + L_2 S_{3e_3}) + v_P (L_1 C_1 + L_2 C_{12}) \right. \\ &\quad \left. + (\omega_E e_1 - e_2 k_2) (L_1 C_{23} + L_2 C_3) \right. \\ &\quad \left. - (L_1 S_{12} + L_2 S_3) (L_3 e_3 (k_3 + \omega_E) + L_3 \omega_R + e_1 k_1 + e_2 \omega_E) \right) \\ \dot{\theta}_3 &= \frac{1}{L_2 S_2} \left( v_R S_{23e_3} + (e_2 k_2 - e_1 \omega_E) C_{23} - v_P C_1 \right. \\ &\quad \left. + (e_2 \omega_E + k_1 e_1 + L_3 (\omega_R + e_3 (k_3 + \omega_E))) S_{23} \right) \\ &\quad + (\omega_R + e_3 (k_3 + \omega_E)) \end{aligned}$$

where  $S_{23e_3} = \sin(\theta_2 + \theta_3 + e_3)$  and  $S_{3e_3} = \sin(\theta_3 + e_3)$ .

When the length of link  $L_i$  is known not exactly, an adaptive controller is designed to attain the control objective by using the estimated value of  $L_i$ .

Let us define the estimation error vector as follows

$$\tilde{L} = L - \hat{L} \quad (14)$$

where  $L = [L_1 \ L_2 \ L_3]^T$ , and  $\hat{L}$  is the estimated value of  $L$ . Now, (1) becomes

$${}^1 \hat{V}_E = \hat{J} \dot{\theta}_d \quad (15)$$

where  $\dot{\theta}_d = [\dot{\theta}_{1d} \ \dot{\theta}_{2d} \ \dot{\theta}_{3d}]^T$  with  $\dot{\theta}_{id}$  is the desired value of  $\dot{\theta}_i$ ,  $\hat{J}$  is estimated matrix of  $J$  as follows:

$$\hat{J} = f(\theta_i, \hat{L}_i) = J - \tilde{J} \quad (16)$$

where

$$\tilde{J} = \begin{bmatrix} -\tilde{L}_3 S_{123} - \tilde{L}_2 S_{12} - \tilde{L}_1 S_1 & -\tilde{L}_3 S_{123} - \tilde{L}_2 S_{12} & -\tilde{L}_3 S_{123} \\ \tilde{L}_3 C_{123} + \tilde{L}_2 C_{12} + \tilde{L}_1 C_1 & \tilde{L}_3 C_{123} + \tilde{L}_2 C_{12} & \tilde{L}_3 C_{123} \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

Substituting (16) into (15) yields

$${}^1 \hat{V}_E = (J - \tilde{J}) \dot{\theta}_d \quad (18)$$

If  $L_i$  is estimated, the position vector of the end-effector becomes

$${}^1 \hat{p}_E = {}^1 p_E - {}^1 \tilde{p}_E \quad (19)$$

$${}^1 \tilde{p}_E = \begin{bmatrix} \tilde{L}_1 \cos(\theta_1) + \tilde{L}_2 \cos(\theta_1 + \theta_2) + \tilde{L}_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ \tilde{L}_1 \sin(\theta_1) + \tilde{L}_2 \sin(\theta_1 + \theta_2) + \tilde{L}_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ 0 \end{bmatrix} \quad (20)$$

Now, (8) can be rewritten as

$$\begin{aligned} \dot{E}_E &= \dot{A}A^{-1}E_E \\ &+ A(V_R - (V_P + W_P \times {}^0\text{Rot}_1^1 \hat{p}_E + {}^0\text{Rot}_1^1 \hat{J}\dot{\theta}_d)) \end{aligned} \quad (21)$$

Substituting (18) and (19) into (21) yields

$$\begin{aligned} \dot{E}_E &= \dot{A}A^{-1}E_E + A(V_R - (V_P + W_P \times {}^0\text{Rot}_1^1 p_E \\ &+ {}^0\text{Rot}_1^1 J\dot{\theta}_d)) + A(W_P \times {}^0\text{Rot}_1^1 \tilde{p}_E + {}^0\text{Rot}_1^1 \tilde{J}\dot{\theta}_d) \end{aligned} \quad (22)$$

$A W_P \times {}^0\text{Rot}_1^1 \tilde{p}_E$  and  $A {}^0\text{Rot}_1^1 \tilde{J}\dot{\theta}_d$  can be rewritten as follows

$$\begin{aligned} A W_P \times {}^0\text{Rot}_1^1 \tilde{p}_E &= A_P \tilde{L} \\ &= \omega_P \overbrace{\begin{bmatrix} -\cos(\theta_2 + \theta_3) & -\cos(\theta_3) & -1 \\ \sin(\theta_2 + \theta_3) & \sin(\theta_3) & 0 \\ 0 & 0 & 0 \end{bmatrix}}^{A_P} \begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \tilde{L}_3 \end{bmatrix} \end{aligned} \quad (23)$$

$$A {}^0\text{Rot}_1^1 \tilde{J}\dot{\theta}_d = A_J \tilde{L}$$

$$= \overbrace{\begin{bmatrix} -\dot{\theta}_1 \cos(\theta_2 + \theta_3) & -(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_3) & -(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \\ \dot{\theta}_1 \sin(\theta_2 + \theta_3) & (\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_3) & 0 \\ 0 & 0 & 0 \end{bmatrix}}^{A_J} \begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \tilde{L}_3 \end{bmatrix} \quad (24)$$

Equation (22) can be re-expressed as follows

$$\begin{aligned} \dot{E}_E &= \dot{A}A^{-1}E_E + A(V_R - (V_P + W_P \times {}^0\text{Rot}_1^1 p_E \\ &+ {}^0\text{Rot}_1^1 J\dot{\theta}_d)) + (A_P + A_J) \tilde{L} \end{aligned} \quad (25)$$

The chosen Lyapunov function and its derivative are

$$V_1 = \frac{1}{2} E_E^T E_E + \frac{1}{2} \tilde{L}^T \gamma \tilde{L} \quad (26)$$

$$\dot{V}_1 = E_E^T \dot{E}_E + \tilde{L}^T \dot{\gamma} \tilde{L} = E_E^T \dot{E}_E - \dot{\tilde{L}}^T \gamma \tilde{L} \quad (27)$$

where  $\gamma = \text{diag}(\gamma_1 \gamma_2 \gamma_3)$  with  $\gamma_i$  is the positive definite value and  $\dot{\tilde{L}}^T = -\dot{\tilde{L}}^T$ .

Substituting (25) into (27) yields

$$\dot{V}_1 = \dot{V}_0 + E_E^T (A_P + A_J) \tilde{L} - \dot{\tilde{L}}^T \gamma \tilde{L} \quad (28)$$

To achieve the negative value of  $\dot{V}_1$ , the control law is chosen as (13), and the parameter update rule is chosen as

$$\dot{\tilde{L}}^T = E_E^T (A_P + A_J) \gamma^{-1} \quad (29)$$

#### a. Controller Design for Mobile Platform

The task of the mobile platform is to move so as to avoid the singularity of the manipulator's configuration. A simple algorithm is proposed for the mobile-platform to avoid the singularity by keeping its initial configuration throughout the welding process.

The initial configuration of the manipulator is chosen as shown in Fig. 3. Let us define a point  $M(X_M, Y_M)$  that coincides to the point  $E$  of the end-effector at beginning. If  $M$  and  $E$  do not coincide, the errors exist. The error vector  $E_M = [e_4 \ e_5 \ e_6]^T$  is defined as

$$E_M = \begin{bmatrix} e_4 \\ e_5 \\ e_6 \end{bmatrix} = \begin{bmatrix} \cos \Phi_M & \sin \Phi_M & 0 \\ -\sin \Phi_M & \cos \Phi_M & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_E - X_M \\ Y_E - Y_M \\ \Phi_E - \Phi_M \end{bmatrix} \quad (30)$$

In order to keep the configuration of the manipulator away from the singularity, the mobile-platform has to move so that the point  $M$  tracks to the point  $E$ . Consequently, the initial configuration of the manipulator is maintained throughout the welding process, and the singularity does not appear. The initial configuration of the manipulator is chosen as  $\theta_1 = \frac{3\pi}{4}$ ,  $\theta_2 = -\frac{\pi}{2}$ ,  $\theta_3 = \frac{\pi}{4}$ . From Fig. 4, we get the geometric relations as

$$\begin{aligned} X_M &= X_P - D \sin \Phi_P, & Y_M &= Y_P + D \cos \Phi_P, \\ \Phi_M &= \Phi_P \end{aligned} \quad (31)$$

where  $D = L_1 \sin\left(\frac{3\pi}{4}\right) + L_2 \sin\left(\frac{3\pi}{4} + \left(-\frac{\pi}{2}\right)\right) + L_3 \sin\left(\frac{3\pi}{4} + \left(-\frac{\pi}{2}\right) + \frac{\pi}{4}\right)$  Hence, we have

$$\begin{aligned} \dot{X}_M &= v_P \cos \Phi_P - D \omega_P \cos \Phi_P, \\ \dot{Y}_M &= v_P \sin \Phi_P - D \omega_P \sin \Phi_P, & \dot{\Phi}_M &= \omega_P \end{aligned} \quad (32)$$

A controller for the mobile-platform will be designed to achieve  $e_i \rightarrow 0$  when  $t \rightarrow \infty$ .

The derivative of (30) can be re-expressed as follows

$$\begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} v_E \cos e_6 \\ v_E \sin e_6 \\ \omega_E \end{bmatrix} + \begin{bmatrix} -1 & e_5 + D \\ 0 & -e_4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_P \\ \omega_P \end{bmatrix} \quad (33)$$

The chosen Lyapunov function and its derivative are

$$V_0 = \frac{1}{2} e_4^2 + \frac{1}{2} e_5^2 + \frac{1 - \cos e_6}{k_5} \quad (34)$$

$$\dot{V}_0 = e_4 \dot{e}_4 + e_5 \dot{e}_5 + \frac{\sin e_6}{k_5} \dot{e}_6$$

$$\begin{aligned} \dot{V}_0 &= e_4(-v_P + D \omega_P + v_E \cos e_6) \\ &+ \frac{\sin e_6}{k_5} (-\omega_P + \omega_E + k_5 e_5 v_E) \end{aligned} \quad (35)$$

An obvious way to achieve negativeness of  $\dot{V}_2$  is to choose  $(v_P, \omega_P)$  as

$$v_P = D \omega_P + v_E \cos e_6 + k_4 e_4 \quad (36)$$

$$\omega_P = \omega_E + k_5 e_5 v_E + k_6 \sin e_6 \quad (37)$$

where  $k_4, k_5,$  and  $k_6$  are positive values.

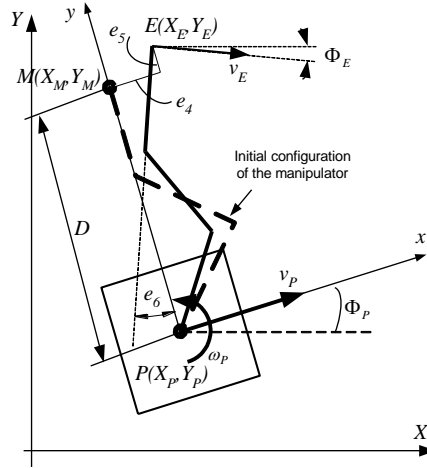


Fig. 3

Scheme for deriving the error equations of mobile-platform

When the wheel radius  $r$  and the distance from the center point to the wheel  $b$  are unknown, we design an adaptive controller by using the estimated values of  $r$  and  $b$ .

Substituting (3) into (33) yields

$$\begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} v_E \cos e_6 \\ v_E \sin e_6 \\ \omega_E \end{bmatrix} + \begin{bmatrix} -1 & e_5 + D \\ 0 & -e_4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} \quad (38)$$

Let us define  $a_1 = \frac{1}{r}$  and  $a_2 = \frac{b}{r}$ . Equation (38) becomes

$$\begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} v_E \cos e_6 \\ v_E \sin e_6 \\ \omega_E \end{bmatrix} + \begin{bmatrix} -1 & e_5 + D \\ 0 & -e_4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2a_1} & \frac{1}{2a_1} \\ \frac{1}{2a_2} & -\frac{1}{2a_2} \end{bmatrix} \begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} \quad (39)$$

Equation (4) becomes

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix} \begin{bmatrix} v_P \\ \omega_P \end{bmatrix} \quad (40)$$

If  $r$  and  $b$  are unknown, (40) becomes

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 \\ \hat{a}_1 & -\hat{a}_2 \end{bmatrix} \begin{bmatrix} v_{Pd} \\ \omega_{Pd} \end{bmatrix} \quad (41)$$

where  $v_{Pd}$  and  $\omega_{Pd}$  are the desired values of  $v_P$  and  $\omega_P$ ,  $\hat{a}_1 = \frac{1}{\hat{r}}$  and  $\hat{a}_2 = \frac{\hat{b}}{\hat{r}}$  are the estimated values of  $a_1$  and  $a_2$ . Substituting (41) into (39) yields

$$\begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} v_E \cos e_6 \\ v_E \sin e_6 \\ \omega_E \end{bmatrix} + \begin{bmatrix} -1 & e_5 + \hat{D} \\ 0 & -e_4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{a}_1 & 0 \\ a_1 & \hat{a}_2 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} v_{Pd} \\ \omega_{Pd} \end{bmatrix} \quad (42)$$

where  $\hat{D} = \hat{L}_1 \sin\left(\frac{3\pi}{4}\right) + \hat{L}_2 \sin\left(\frac{3\pi}{4} + \left(-\frac{\pi}{2}\right)\right) + \hat{L}_3 \sin\left(\frac{3\pi}{4} + \left(-\frac{\pi}{2}\right) + \frac{\pi}{4}\right)$

$$v_{Pd} = \frac{a_1}{\hat{a}_1} v_P = \hat{r} a_1 v_P = \frac{\hat{r}}{r} v_P, \quad ,$$

$$\omega_{Pd} = \frac{a_2}{\hat{a}_2} \omega_P = \frac{\hat{r}}{\hat{b}} a_2 \omega_P = \frac{\hat{r}}{\hat{b}} \frac{b}{r} \omega_P$$

Let us define the estimation errors as follows

$$\tilde{a}_1 = a_1 - \hat{a}_1, \quad \tilde{a}_2 = a_2 - \hat{a}_2 \quad (43)$$

Equation (42) can be rewritten as follows

$$\begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} v_E \cos e_6 \\ v_E \sin e_6 \\ \omega_E \end{bmatrix} + \begin{bmatrix} -1 & e_5 + \hat{D} \\ 0 & -e_4 \\ 0 & -1 \end{bmatrix} \left( \begin{bmatrix} v_{Pd} \\ \omega_{Pd} \end{bmatrix} - \begin{bmatrix} \tilde{a}_1 v_{Pd} \\ a_1 \\ \tilde{a}_2 \omega_{Pd} \\ a_2 \end{bmatrix} \right) \quad (44)$$

The chosen Lyapunov function and its derivative are given as

$$V_3 = \frac{1}{2}e_4^2 + \frac{1}{2}e_5^2 + \frac{1}{2}e_6^2 + \frac{1}{2\gamma_{P1}a_1}\tilde{a}_1^2 + \frac{1}{2\gamma_{P2}a_2}\tilde{a}_2^2 \quad (45)$$

$$\begin{aligned} \dot{V}_3 &= e_4\dot{e}_4 + e_5\dot{e}_5 + e_6\dot{e}_6 + \frac{1}{\gamma_{P1}a_1}\tilde{a}_1\dot{\tilde{a}}_1 + \frac{1}{\gamma_{P2}a_2}\tilde{a}_2\dot{\tilde{a}}_2 \\ &= \dot{V}_2 + \frac{\tilde{a}_1}{a_1} \left( e_4 v_{Pd} - \frac{1}{\gamma_{P1}} \dot{\tilde{a}}_1 \right) + \frac{\tilde{a}_2}{a_2} \left( e_6 \omega_{Pd} - \frac{1}{\gamma_{P2}} \dot{\tilde{a}}_2 - e_4 \hat{D} \omega_{Pd} \right) \end{aligned} \quad (46)$$

where  $\dot{\tilde{a}}_1 = -\dot{\hat{a}}_1$  and  $\dot{\tilde{a}}_2 = -\dot{\hat{a}}_2$ .

The controller is still (36) and (37), but there is two update laws as follows

$$\dot{\hat{a}}_1 = \gamma_{P1} e_4 v_{Pd} \quad (47)$$

$$\dot{\hat{a}}_2 = \gamma_{P2} \omega_{Pd} (e_6 - e_4 \hat{D}) \quad (48)$$

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

Table 1 shows the parameter and initial values for the welding wheeled mobile manipulator system used in this simulation.

Table 1  
Numerical values and initial values for simulation

| Parameters              | Value<br>s | Units | Parameters       | Value<br>s | Units              |
|-------------------------|------------|-------|------------------|------------|--------------------|
| $X_R - X_E (t=0)$       | 0.005      | m     | $k_1$            | 1.4        | /s                 |
| $Y_R - Y_E (t=0)$       | 0.005      | m     | $k_2$            | 1.5        | /s                 |
| $\Phi_R - \Phi_E (t=0)$ | 15         | deg.  | $k_3$            | 1.7        | /s                 |
| $X_E (t=0)$             | 0.275      | m     | $\theta_1 (t=0)$ | 135        | deg.               |
| $Y_E (t=0)$             | 0.395      | m     | $\theta_2 (t=0)$ | -90        | <b>deg.</b>        |
| $\Phi_E (t=0)$          | -15        | deg   | $\theta_3 (t=0)$ | 45         | <b>deg.</b>        |
| $v_R$                   | 0.007<br>5 | m/s   | $k_5$            | 65         | rad/m <sup>2</sup> |
| $k_4$                   | 1.6        | /s    | $k_6$            | 1.25       | rad/s              |

In this case, the update laws in (29), (47) and (48) are used to estimate the unknown parameters. Parameter values for simulation are shown in Table 2.

A welding mobile manipulator of prototype as shown in Fig. 4 has constructed to check the effectiveness of the algorithm.



Table 2  
The parameter values for simulation

| Parameters       | Value<br>s | Units | Parameters    | Value<br>s | Units |
|------------------|------------|-------|---------------|------------|-------|
| $\hat{b}(t=0)$   | 0.11       | m     | $\gamma_1$    | 0.265      |       |
| $\hat{r}(t=0)$   | 0.027      | m     | $\gamma_2$    | 0.187<br>5 |       |
| $\hat{L}_1(t=0)$ | 0.195      | m     | $\gamma_3$    | 0.163<br>5 |       |
| $\hat{L}_2(t=0)$ | 0.195      | m     | $\gamma_{P1}$ | 4640       |       |
| $\hat{L}_3(t=0)$ | 0.195      | m     | $\gamma_{P2}$ | 12.4       |       |

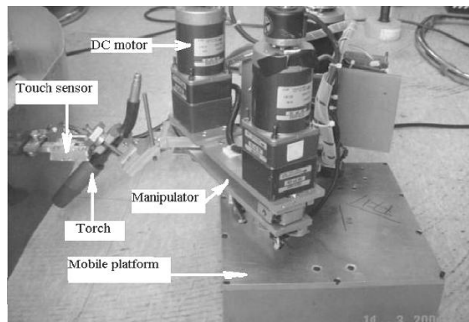


Fig. 4  
The wheeled mobile manipulator prototype

The simulation and experimental results of the system with all exactly unknown parameters are presented as the following figures 5-11.

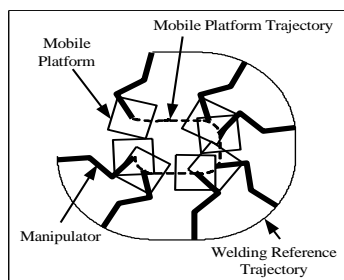


Fig. 5  
The trajectory reference and mobile manipulator posture

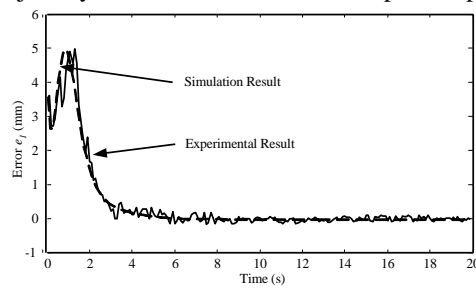


Fig. 6  
Tracking error  $e_1$  from the simulation and experimental results

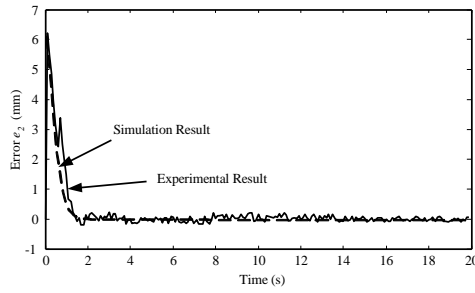


Fig. 7

Tracking error  $e_2$  from the simulation and experimental results

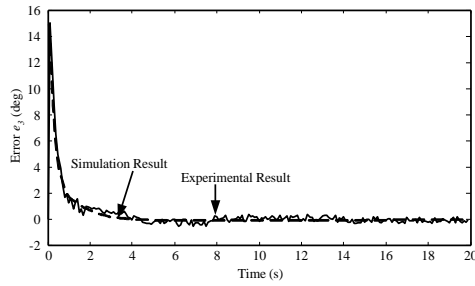


Fig. 8

Tracking error  $e_3$  from the simulation and experimental results

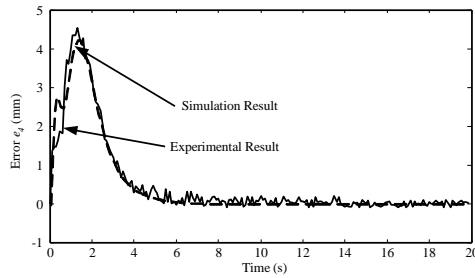


Fig. 9

Tracking error  $e_4$  from the simulation and experimental results

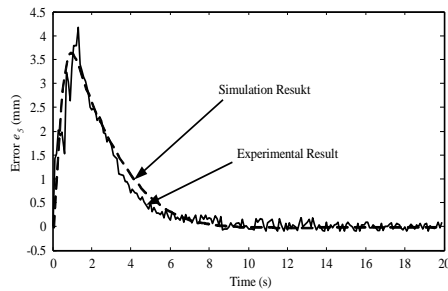


Fig. 10

Tracking error  $e_5$  from the simulation and experimental results

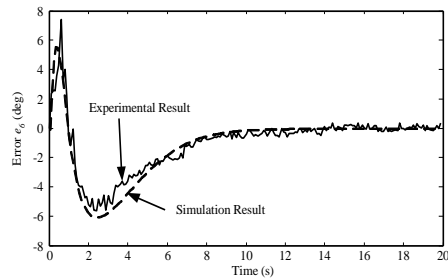


Fig. 11

Tracking error  $e_6$  from the simulation and experimental results

## V. CONCLUSIONS

This paper proposed an adaptive tracking control method for the mobile platform with the unknown parameters. Two independent controllers are proposed to control two subsystems. The controllers are based on the Lyapunov function to guarantee the tracking stability of the welding mobile robot. The dimensional parameters of the mobile manipulator are considered to be unknown parameters which are estimated by using update law in adaptive control scheme. The simulation and experiment results show that all of the errors are converged. The controllers are simple. Those are implemented for microcontroller easily.

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