

## Characterization of Polymeric Membranes under Large Deformations by a Neural Model

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**Abstract:** Recent progress in polymers modeling shows a need for a precise description of the material behavior under the combined effect of the applied efforts and temperature. The Artificial Neural Networks (ANN), whose operation is based on some properties of biological neurons, contain various architectures strongly interrelated treating elements and offering an alternative to conventional computational approaches. They respond in parallel to a set of inputs, and are affected by transformations more than by algorithms and procedures. They can achieve complicated operations of Input-Output without explicit relationships (linear and nonlinear) between all the presented data during a learning process. In our work, we are interested by the characterization of circular thermoplastic membranes, ABS (Acrylonitrile butadiene styrene), under biaxial deformation using the free blowing technique. Hyperelastic Mooney-Rivlin model is considered. A neural algorithm (based on artificial neural networks) is used to model the behavior of blowing these membranes, the results are compared with experimental studies and numerical finite differences model.

**Key words :** membrane, Artificial Neural Networks, free blowing, hyperelasticity, ABS.

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### I. INTRODUCTION

Working a polymeric sheet attached by its contour in a 3D shape is the main feature of the process of blow molding. Generally, the deformation is very fast, non-uniform, multiaxial, and occurs at a forming temperature above the glass transition temperature [1]. The modeling of this phenomenon remains delicate and creates several difficulties as in the experimentation level [2] as in the modeling level in terms of important computational time, as well as the implementation of nonlinear constitutive laws...

Often, as the bubble size increases and the internal pressure approach a maximum value beyond it there no solution, the numerical scheme diverges. So, to remedy this problem, we consider in this work, an approach based on an artificial neural networks model [3], to model the behavior of thermoplastic membranes. Recently, neural networks are used in many areas include the classification, pattern recognition, speech synthesis, diagnosis, identification and control of dynamic systems, or even to improve the quality of produced parts in various industrial processes such as injection molding or blow molding. The use of neural networks in the identification processes is justified by the fact that they can be well approximated nonlinear functions through their capacity to reproduce rather complex behavior, and their performances are improving so continuously while relying on a dynamic learning [4], which provides a robust neuronal identification vis-à-vis the parametric variations and disturbances that can affect the operation of the system studied. Modeling using neural networks requires among a model selection which is a crucial step in designing a neural network, as with any non-linear model. Indeed, this phase should lead to choose the model that has a complexity that is both sufficient to fit the data and not excessive. In this work, the power of neural networks is used to model the nonlinear behavior in free bowing of a polymeric membrane in ABS. A multilayer neural network is used for this application, in order to reproduce the behavior of this membrane; the results are compared with experimentation as well as with finite difference model [5].

### II. ARTIFICIAL NEURAL NETWORKS

#### 2.1 Introduction

Neural networks have been studied since the 40's with the notion of formal neuron of McCulloch and Pitts [6] who first developed a mathematical model of biological neuron. Hence a neural network is a mathematical model of treatment consists of several basic interconnected neurons via weights that play the role

of synapses. These weights assume values that define the behavior of the whole structure. The adaptation of the latter through a suitable learning algorithm, allows the network to learn, memorize and generalize information.

## 2.2 Multilayer network

### 2.2 Architecture

In general, a multilayer network is composed of an input layer, an output layer, and a number of hidden layers. Each neuron is connected to all neurons of the next layer via connections whose weights are real. Usually, the neuron used in the hidden layers or in the output layer, performs a weighted summation of its inputs and a nonlinear transformation which is often the sigmoid function. While the neuron used in the input layer, simply passes its input to output.

The architecture of a multilayer network can be given by the following figure, Fig 1 :

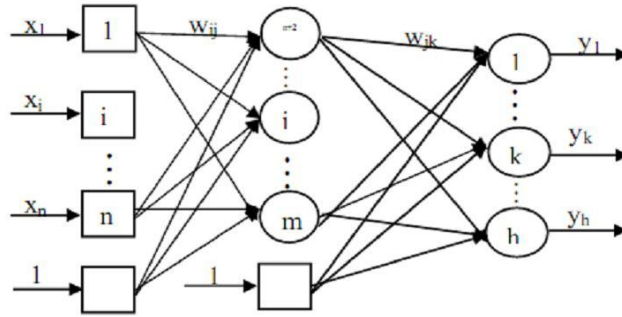


Figure 1. Architecture of a multilayer perceptron

### 2.3 Learning process

The learning of a neural network means that it changes its behavior to allow it to move towards a clearly defined purpose. In other words, learning is adjusting the connection weights so that the outputs of the network are as close as possible to the desired outputs for all the used examples. Initially, the learning of multilayer networks was determined by the gradient method. Then, all optimization methods have been applied to this problem as the conjugate gradient method, the Levenberg-Marquardt [7,8] or Kalman filters... Thus, the literature on learning of multilayer neural networks is very rich. However, we limit the study of the back-propagation algorithm combined with a Levenberg-Marquardt optimization algorithm, which is a modified version of the gradient method and remains the most used and most responded. This is an optimization algorithm that seeks to minimize a cost function which covers the gap between the desired output and that delivered by the network, to expand more knowledge, we bring the reader to references [4] and [9].

The back-propagation algorithm can be described using the following two phases: Propagation phase or relaxation: initially, the input values are provided to the network's input layer, then the network propagates the information from one layer to another to the output layer, and which gives the response of network.

Back-propagation phase: that is to calculate the difference between the outputs of the network and the desired resulting changes in weight of the output neurons. Then, retro-propagating the error to the input layer which allows the adaptation of the weights of the hidden layer.

In our network, we used a learning algorithm by back-propagation associated with a Levenberg-Marquardt optimization algorithm [9], with a transfer function of sigmoid type.

In our architecture, the error in the presentation of an example, denoted by E, can be given by the following expression:

$$E = 1/2 \sum_{k=1}^h (Y_k - Y_k^d)^2 \quad (1)$$

wherein,  $Y_k^d$  represents the  $K^{th}$  component of the desired output vector. The overall error to minimize is the sum of all errors on the database. The inputs and outputs of the neurons j and k explicit from the following expressions:

$$E_j = \sum_{i=1}^{n+1} x_i w_{ij} \quad ; \quad s_j = f(E_j) \quad (2)$$

$$E_k = \sum_{j=1}^{m+1} s_j w_{jk} \quad ; \quad Y_k = f(E_k) \quad (3)$$

Where  $f$  is the sigmoid activation function defined by:

$$f(x) = \frac{1}{1 + e^{-x}} \quad (4)$$

The adjustment of the weights of the network by the back-propagation algorithm is ensured through the following iterative equation:

$$w_{uv}(i) = w_{uv}(i - 1) - \varepsilon \frac{\partial E}{\partial w_{uv}} \quad (5)$$

With  $\varepsilon$  and  $i$  denote the pitch and the iteration number.

The derivative of  $E$  with the weight depends on its position, in fact:

If the weight connects the hidden layer to the output layer, so:

$$\frac{\partial E}{\partial w_{jk}} = f'(E_k)(Y_k - Y_k^d)s_j \quad (6)$$

If the weight connects the input layer to the hidden layer, then:

$$\frac{\partial E}{\partial w_{jk}} = f'(E_j) \left[ \sum_{k=1}^h f'(E_k)(Y_k - Y_k^d) \right] x_i \quad (7)$$

It should be emphasized that the convergence of the back-propagation algorithm depends on a variety of parameters, namely:

The complexity of the database and the order of presentation of examples. Indeed, various studies have shown that a random sample presentation allows faster learning, The structure of the neural network considered and especially the number of neurons in the hidden layer which must not be very important to ensure the generalization ability of the network, or very small for the network to learn, The initial setting of the algorithm parameters, essentially the initial values of weight and iteration pitch must be chosen in a manner allowing to have a fast convergence.

The multilayer neural network that we propose has the following structure:

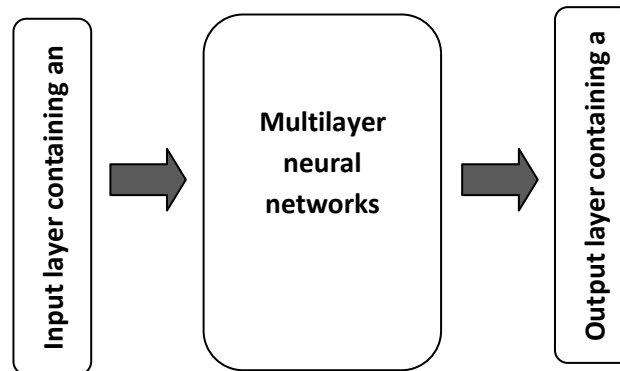


Figure 2. Structure of the used multilayer neural network

The used network consists of an input layer, hidden layers, and an output layer. The number of neurons in the output layer depends on the response contemplated by the network. For the identification program, two neurons are required in the output layer to give the response of the excitation, while the input of the network is the variance of the corresponding pressures. As for modeling behavior of the membrane, the neuron will output variance in response to pressure, while the input will be presented by the stretch ratios. The experimental data are used for network learning, also, the back-propagation algorithm is used for error of perceptron learning. Thus, the optimization algorithm of Levenberg-Marquardt is used also in view of minimizing the error during the operation of back propagation, this optimization technique is more powerful than other optimization algorithms, but it requires more memory.

### III. EXPERIMENTAL

The materials considered in this work is the ABS (Acrylonitrile butadiene styrene). The initial sheet thickness was 0.46mm. The exposed circular domain of radius  $R=3.175$  cm is heated to the softening point inside a heating chamber using infrared heaters. When the temperature was quite uniform over the flat sheet, the inflation was started using compressed air at a controlled flow rate. In most inflation tests the experiment ended when the bubble burst. The bubble pressure, its height at the pole and time are recorded simultaneously using a video camera and a data acquisition system.

### IV. RESULTS

Fig 3 below shows a comparison between our neural model, finite difference model using a hyperelastic Ogden model [10, 11], and experimental measurements.

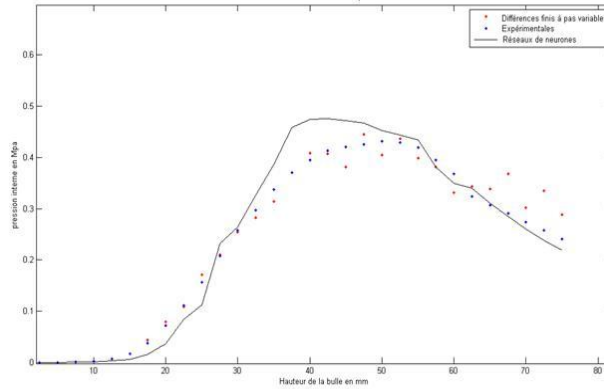


Figure 3. Internal pressure versus the bubble height, comparison between experimental results, finite difference model, and neural networks model

Note that the neural model we propose reproduces faithfully the behavior of the ABS membrane as well as the finite difference model. The margin of error between the neural model and the experimental curve remains negligible, and can also be minimized while depending on the choice of the number of hidden layers in the network and the chosen optimization algorithm. By increasing the number of hidden layers in the network, while keeping the same transfer function and the same optimization algorithm of Levenberg-Marquardt, the neuronal model can be adjusted for more fitting with the experimental results,

Fig 4 below shows the results of model fitting:

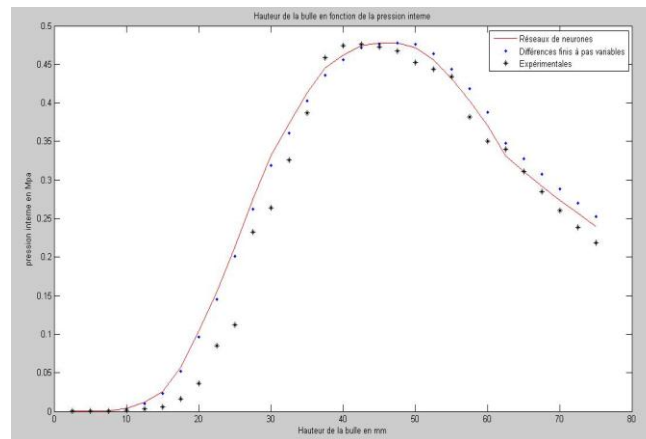


Figure 4. Internal pressure versus the bubble height, comparison between experimental results, finite difference model, and neural networks model (Adjusted model)

It is clear that the neural model that we propose reproduces the behavior of the membrane, but, the fitting procedure that contains a number of algorithms, affects the computation time which becomes increasingly important also with increasing the number of hidden layers in the network, which induces the addition of other conditions nonlinearity.

## V. CONCLUSION

The objective of this work is to characterize the hyperelastic behavior of thermoplastic sheets used in the process of thermoforming, blow molding or injection blow molding, with other modeling methods than conventional methods, by exploiting the robustness of neuronal models. The results have accentuated the strength of the model chosen to reproduce the behavior of membranes, regardless of material or geometric nonlinearities presented in the study of such materials. A conclusion section must be included and should indicate clearly the advantages, limitations, and possible applications of the paper. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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