

## Pitch Attitude Control of a Booster Rocket

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**Abstract**-There are three important variables for a rocket moving through air and then through space. These are its yaw, pitch and roll. These variables are to be precisely controlled in order to reach the target and produce the desired effect. The paper deals with pitch attitude control of a booster rocket for launching a space-ship. Starting from its block diagram, the mathematical model has been developed and the analytical treatment has been made by indigenously made computer program and also by using MATLAB-tools for getting the desired results. Fulfillment of the specifications could be made suitably choosing the forward path gain and the coefficient of rate feedback.

**Keywords**-Feedback, Overshoot, Phase margin, Pitch attitude, Rocket

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### I. INTRODUCTION

A rocket is essential for space travel. It is a self-propelled device that carries its own fuel, as well as oxygen, or other chemical agent, needed to burn its fuel. Most rockets move by burning their fuel and expelling the hot exhaust gases that result. The force of these hot gases shooting out in one direction causes the rocket to move in the opposite direction. A rocket engine is the most powerful engine for its weight. Other forms of propulsion, such as jet-powered and propeller-driven engines, cannot match its power. Rockets can operate in space, because they carry their own oxygen for burning their fuel. Rockets are presently the only vehicles that can launch into and move around in space [17,18].

Space exploration is a challenging activity due to its complexity. The complication arises out of so many obstacles which we are to overcome [1,2,3,4,5] e.g.

- The vacuum of space
- Heat management problems
- The difficulty of re-entry
- Orbital mechanics
- Micrometeorites and space debris
- Cosmic and solar radiation
- The logistics of having restroom facilities in a weightless environment

But the biggest problem of all is harnessing enough energy simply to get a spaceship off the ground. This is possible due to high capacity of rocket engines. The rocket engines are easy to build for flying inexpensively. But that is true only for the toy level. For propulsion through space it is so highly complicated [9,10] that space endeavours have been taken up only by some advanced countries of the world. India is one amongst the successful countries in space endeavours. Our ISRO is one of the pioneers in space activity.

### II. PRINCIPLE OF A ROCKET ENGINE

Rocket engines are reaction engines [17,18]. The basic principle is the Newton's 3<sup>rd</sup> law i.e. action is equal to the reaction. A rocket engine throws mass in one direction and moves in the opposite direction by reactionary forces. The "strength" of a rocket engine is called its thrust. It is generated by throwing burnt fuels backward. The flight can be controlled by changing the rate of burning of the fuel and the direction of throw. Motion along three principal axes is important while it moves through air and subsequently through space. The principal axes are the following:

- Vertical axis, or yaw axis — an axis drawn from top to bottom, and perpendicular to the other two axes.

- Lateral axis, transverse axis, or pitch axis — an axis running from the pilot's left to right in piloted aircraft, and parallel to the wings of a winged aircraft.
- Longitudinal axis, or roll axis — an axis drawn through the body of the vehicle from tail to nose in the normal direction of flight, or the direction the pilot faces. The motion along these three principal axes is to be closely controlled [1,2,3]. This is accomplished by proportional-derivative (PD) feedback control. The derivative or rate feedback as well as the forward path gain are to be adjustable [6,7,12].

### III. THE MATHEMATICAL MODEL AND THE BLOCK DIAGRAM

The pitch attitude control of a booster rocket contains both attitude and rate gyros [6,7,12]. The block diagram [8] representation is given in fig. 1. We are aiming at finding out amplifier gain  $K$ , and the feedback coefficients for best possible dynamic performance. The system should be almost critically damped. The percentage overshoot has to be limited to 0.1% and the phase margin must be at least  $30^\circ$ .

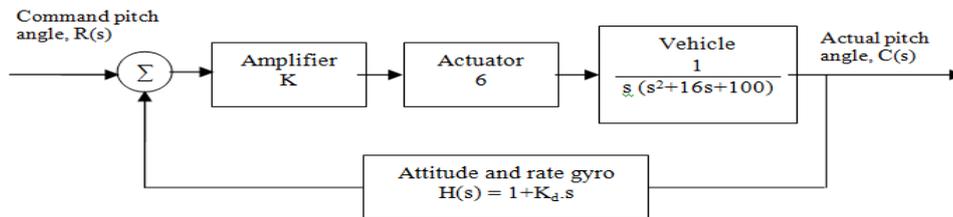


Fig. 1. Pitch attitude control of booster rocket: block diagram

The transfer function of the forward and backward path are given below:

$$G(S) = \frac{6K}{s(s^2 + 16s + 100)}; H(s) = 1 + K_d s$$

1

The closed loop transfer function is given as:

$$M(S) = \frac{6K}{s^3 + 16s^2 + (6KK_d + 100)s + 6K}$$

2

### IV. ANALYTICAL TECHNIQUES

The general practice is to go in for root contour to find out appropriate values of gain and feedback coefficients. But the method is complex. So, we have treated the problem analytically [6,7]. The solution has been obtained by a specially constructed program [14,15,16]. The program finds the characteristic equation choosing values of gain and feedback coefficients arbitrarily. Then a cut and try process has been followed to realize a damping factor of nearly unity for the dominant poles and at the same time a phase margin of atleast  $30^\circ$ . The specifications could not be fulfilled in several attempts. The computer print-out changed to word-format, is given below:

The programme finds out amplifier gain and feedback coefficients of a booster rocket

The chosen parameters:

Amplifier gain,  $K = 87$ ;

Coefficient of attitude feedback,  $K_p = 1$ ; coefficient of rate feedback,  $K_d = 0.2$

Roots of the characteristic equation are being found out by Coates' method (3rd order convergence)[14,15] to reduce the no. of steps.

The characteristic equation is given below:

$$s^3 + 16s^2 + 204.4s + 522 = 0$$

3

1st root: -3.192409; 2nd & 3rd roots: -6.4038 ± j 11.068

The natural frequency of oscillation for the dominant roots = 12.787 r/s

The damping factor = 0.5008

Corresponding damped frequency of oscillation = 11.068 r/s

The percent overshoot = 16.24

The peak time = 0.28384 sec

The settling time = 0.3614 sec

The gain crossover occurs at an angular frequency of 7.3494 r/s

Phase margin = 77.13 °

The Nyquist plot does not intersect the real axis. So the gain margin is infinity. The phase margin is adequate. But the time-domain performance is not acceptable. The percentage overshoot is very high, much above the specified value. Therefore, we make recourse to MATLAB-tools to get an acceptably good design.

## V. MATLAB-BASED ANALYSIS

Now we make recourse to MATLAB-tools for refinement of the design [11,12,13]. We set the damping coefficient arbitrarily at  $K_d = 0.1$  and check the t-domain and f-domain performance. We find that the overshoot is as high as 10.3%. Then we gradually increase the value of  $K_d$  to 0.14 in steps of 0.02 and check the same. The t-domain and f-domain (Bode plot) responses are given in fig. 2 to fig. 7. It is observed that the best results are obtained for a damping coefficient of  $K_d = 0.14$ .

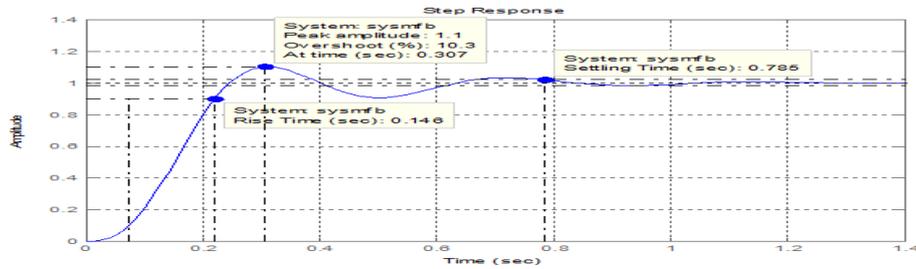


Fig 2 Step response with  $K_d = 0.1$

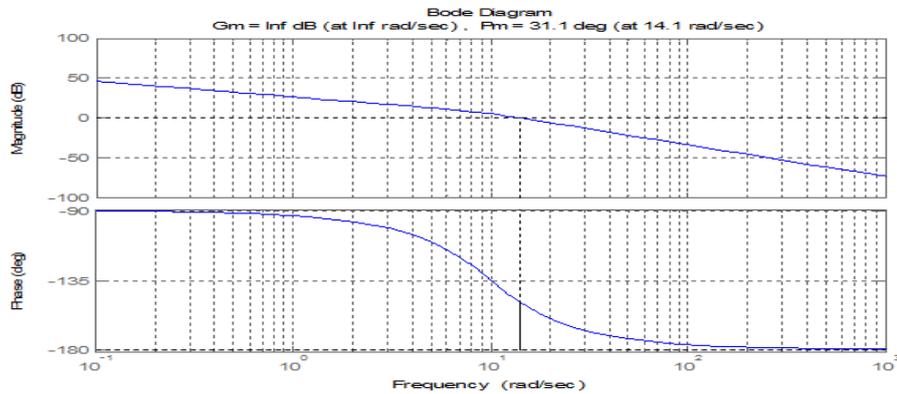


Fig. 3 Bode plot for  $K_d = 0.1$

For  $K_d = 0.1$ , we get a rather high overshoot of 10.3% at a peak time of 0.307 s. The rise time is 0.148 s and the settling time is 0.785 s. The phase margin is  $31.1^\circ$  at 14.1 r/s. This design is not acceptable as the overshoot is high. We want almost critical damping.

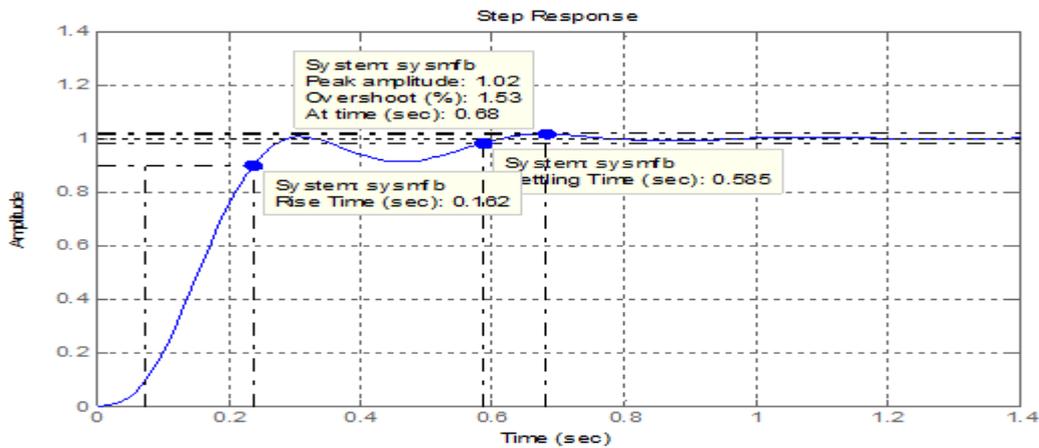


Fig 4 Step response with  $K_d = 0.12$

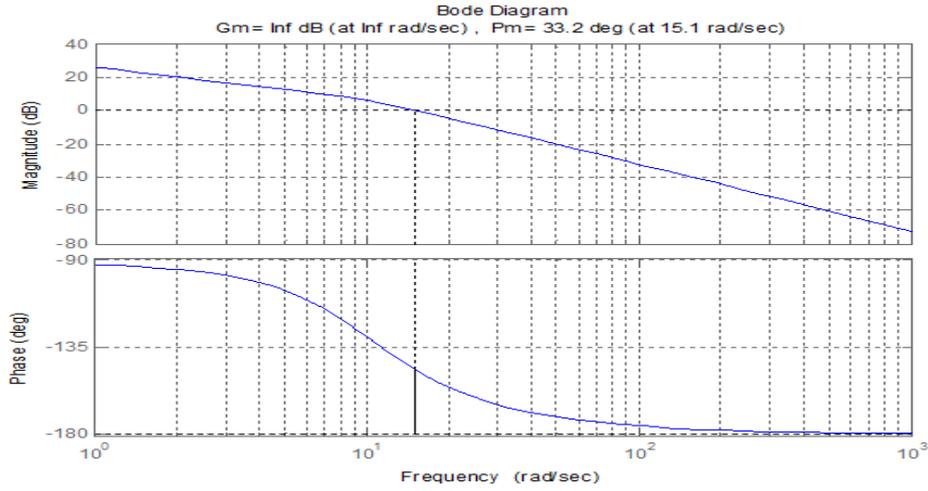


Fig 5 Bode plot with  $K_d = 0.12$

For  $K_d = 0.12$ , we get a much reduced overshoot of 1.53% at a peak time of 0.68 s. The rise time is 0.162 s and the settling time is 0.585 s. The phase margin is  $33.2^\circ$  at 15.1 r/s. This design is much better. But we shall try to further reduce the overshoot and achieve critical damping.

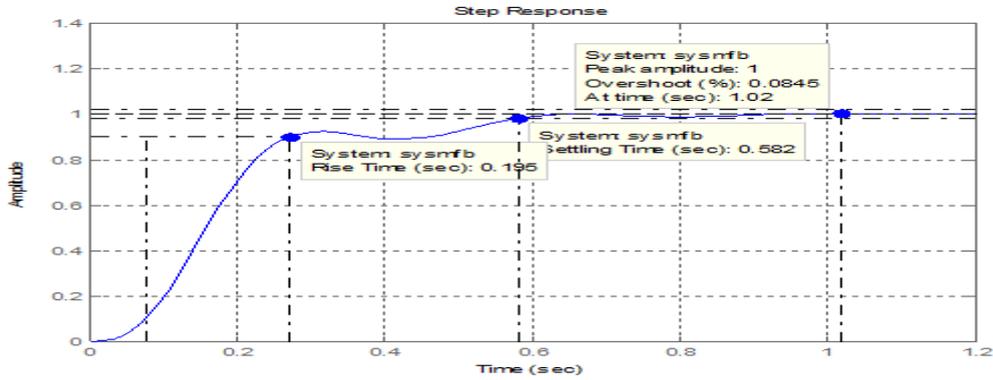


Fig 6 Time domain response for  $K_d = 0.14$

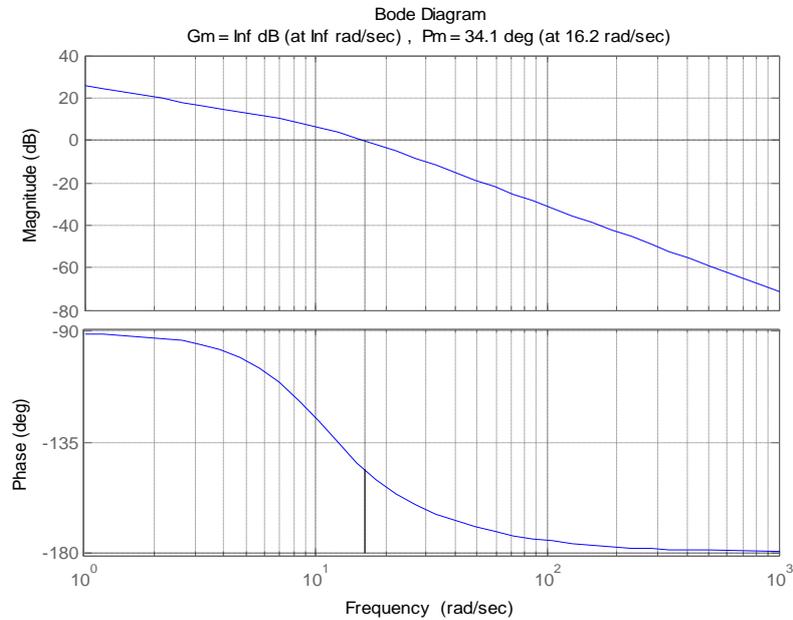


Fig. 7 Bode plot for  $K_d = 0.14$

For  $K_d = 0.14$ , we get an overshoot of 0.0845% at a peak time of 1.02 s. The rise time is 0.195 s and the settling time is 0.582 s. The phase margin is  $34.1^\circ$  at 16.2 r/s. The overshoot is now extremely small. We have almost reached critical damping. We accept the third design as final.

The experimental results for different values of  $K_d$  are given in table 1

**TABLE 1 COMPARISON OF RESULTS (PA CONTROL)**

Damping coeff.	Rise time	Peak time	Peak overshoot	Settling time	Phase margin
$K_d = 0.10$	0.146 s	0.307 s	10.3%	0.785 s	$31.1^\circ$
$K_d = 0.12$	0.162 s	0.68 s	1.53%	0.585 s	$33.2^\circ$
$K_d = 0.14$	0.195	1.02 s	0.0845	0.582	$34.1^\circ$

The gain margin is infinity for all the cases.

## VI. CONCLUSION

A booster rocket is used to launch a space-ship from launch-pad to its spatial flight orasatellite to its desired orbit. The rocket has a powerful engine which operates on the principle Newton's 3<sup>rd</sup>. law. It is propelled controlled combustion of fuel as well as the burning material within the rocket. At first, it moves through air and then through space. The speed has to be continuously controlled as the rocket leaves the launch-pad moves to higher and higher altitude[17,18]. The motion control has to be exercised along three principle axes viz. yaw, pitch and roll. The paper deals with the design and performance evaluation of automatic control system intended forpitch attitude control of the rocket[1]. The control is of proportional-derivative (PD) type, with controllable rate feedback so as to keep the system only very slightly under damped retaining the phase margin above a specified value. It was tried at first to reach the target by analytical treatment and self-made program. But the attempt failed. Then recourse was made to MATLAB-tools [11,12,13]. The specifications could be reached by simultaneous change of the forward path gain and the coefficient of rate feedback.

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