Research Inventy: International Journal Of Engineering And Science
Vol.2, Issue 12 (May 2013), Pp 26-35
Issn(e): 2278-4721, Issn(p):2319-6483, Www.Researchinventy.Com

Improvement in Radiation Pattern Of Yagi-Uda Antenna

[1,]Ankit Agnihotri , [2,]Akshay Prabhu , [3,]Dheerendra Mishra
B.tech (EC), Kanpur Institute of Technology,
A-1 UPSIDC, Industrial Area, Rooma, Kanpur (U.P.) (affiliated to GBTU University, Lucknow),
India

Abstract : *An Antenna is used to transmit and receive electromagnetic waves. Antennas are employed in systems such as radio and television broadcasting, point-to-point radio communication, wireless LAN, radar, and space exploration. Antennas usually work in air or outer space, but can also be operated under water or even through soil and rock at certain frequencies for short distances. The origin of the word antenna relative to wireless apparatus is attributed to Guglielmo Marconi. Several critical parameters affecting an antenna's performance are resonant frequency, impedance, gain, aperture or radiation pattern, polarization, efficiency and bandwidth. Transmit antennas may also have a maximum power rating, and receive antennas differ in their noise rejection properties. We have simulated the radiation pattern of Yagi-Uda antenna in MATLAB. We have designed this antenna and have made improvements in the previous designs to have better electric field intensity and directivity. Our basic approach was to simulate the radiation pattern for a symmetrically shaped antenna and then maximizing the output parameters by using various techniques such as using reflector surfaces wherever the loss in antenna was due to side lobes. Polarization of an antenna is a very important parameter in determining the loss in transmission. In antennas matching plays a very important role in determining the final output and raindrops due to reflection properties can lead to serious weakening of signal at high frequencies, due to which circular polarization is generally preferred.*

***Keywords**: Electromagnetic Waves, Noise Rejection Properties, Reflector Surfaces, Circular Polarization.*

I. INTRODUCTION

A Yagi-Uda Antenna, commonly known simply as a Yagi antenna or Yagi, is a directional antenna system consisting of an array of a dipole and additional closely coupled parasitic elements (usually a reflector and one or more directors).

The geometry of the Yagi-Uda array:

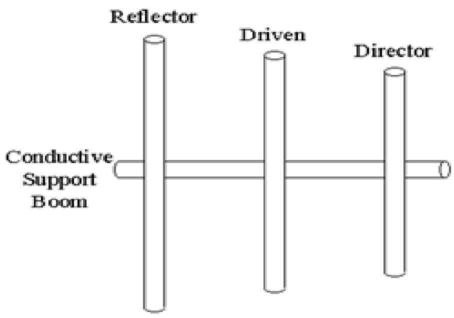

Figure 1.1 Element Yagi-Uda Antenna

The second dipole in the Yagi-Uda array is the only driven element with applied input/output source feed, all the others interact by mutual coupling since receive and reradiate electromagnetic energy; they act as parasitic elements by induced current. It is assumed that an antenna is a passive reciprocal device and then may used either for transmission or for reception of the electromagnetic energy, this well applies to Yagi-Uda also.

II. RADIATION PATTERN

In the field of antenna design the term 'radiation pattern' most commonly refers to the directional (angular) dependence of radiation from the antenna. An antenna radiation pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. Mostly it is determined in the far field region and is a function of directional coordinates. Radiation property is the two or three-dimensional spatial distribution of radiated energy. It may include power flux density, radiation intensity, field strength, directivity or polarization.

The spatial variation of electric or magnetic field is called field pattern. An isotropic radiator is a theoretical point source of waves, which exhibits the same magnitude or properties when measured in all directions. It has no preferred direction of radiation. It radiates uniformly in all directions over a sphere centered on the source.

A directional antenna or beam antenna is an antenna, which radiates greater power in one, or more directions allowing for increased performance on transmit and receive and reduced interference from unwanted sources. Directional antennas like Yagi-Uda antennas provide increased performance over dipole antennas when a greater concentration of radiation in a certain direction is desired. An omni directional antenna is an antenna system, which radiates power uniformly in one plane with a directive pattern shape in a perpendicular plane.

Various parts of a radiation pattern are referred to as lobes, which may be either major, minor, side or back lobes.

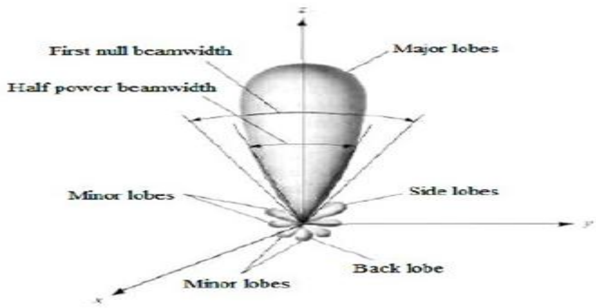

Figure 2.1 Parts Of Radiation Pattern

III. WORKING PRINCIPLE

The simplest or minimal Yagi-Uda antenna has at least two parasitic elements behind the Driven Element (DE); the antenna with only one parasitic element as Reflector element (Ref) is generally called Yagi antenna. This happens when the electrical length of the parasitic element is greater than the driven element.

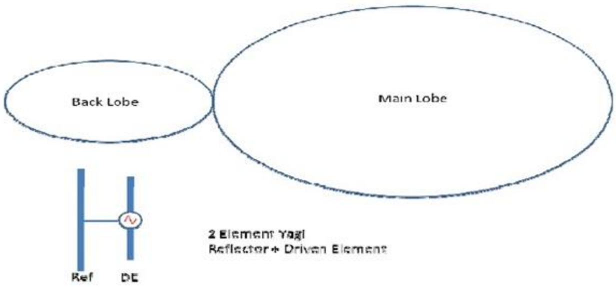

Figure 3.1 Element Yagi (Reflector+Driven Element)

If the electrical length of the parasitic element is shorter than the driven element, the radiation pattern reversed and the parasitic element became a Director (D) always in the two-elements of the Yagi antenna.

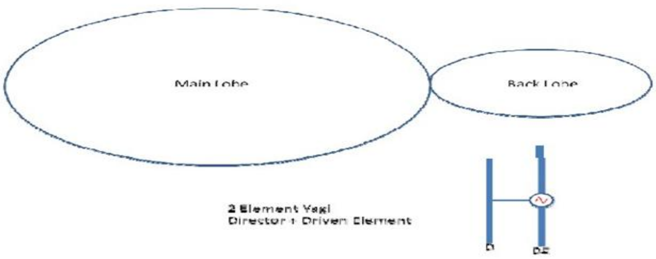

Figure 3.2 Element Yagi (Director+Driven Element)

Then the basic antenna, driven element with both Reflector and Director is called three elements Yagi-Uda, with increased directivity or beam Gain.

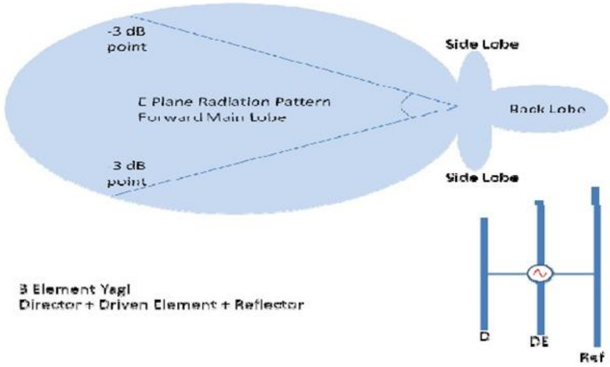

Figure 3.3 Element Yagi (Reflector+Driven Element+Director)

The reflector and directors in the Yagi-Uda antenna are so coupled into parasitic mode; they mutually alter the radiation parameters of the driven element and for each element of the array. Then the physical discovery consist in the increased gain by narrowing the beam width of the dipole alone in a very genially cheap manner, by the means of simple metallic rod or tube conductors, then focus the electromagnetic energy into the desired directions.

More than one parasitic element should be axially added in the front of the driven element and each one is called director. As the reflector, the directors (D1…Dn) has not wired directly to the feed point. As the number of director grows, it increases the directivity as the beam gain of the Yagi-Uda system array.In modern Yagi-Uda design, the parasitic elements should be applied to increase the impedance bandwidth also, much more than a single dipole alone, this is in advance to directional capability of the system to control pattern and impedance with any possible desired combination. Yagi-Uda antennas are widely used in civilian, simple or professionals, military applications also. Yagi-Uda design is used by lot of amateur radio enthusiast all over the world in advance for any kind of wireless radio communication, television etc.

IV. MATHEMATICAL ANALYSIS

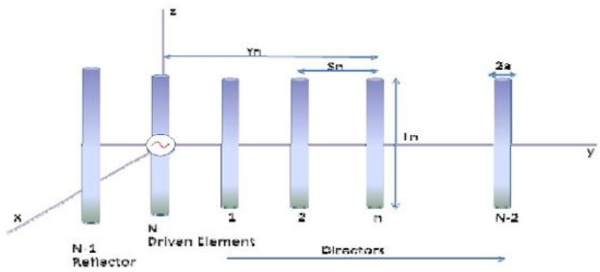

Figure 4.1 Yagi-Uda Antenna's Set Up

The approach taken in formulating the method of solving the Yagi-Uda-type antenna problem is based on an integral equation for the electric field of the array. The point-matching technique is then used to satisfy the integra1 equation at discrete points on the axis of each element rather than attempting to satisfy this equation everywhere on the surface of every element. Thus a system of linear algebraic equations is generated in term of the complex coefficients in the Fourier series expansion of the currents on the elements. Inversion of the matrix yields the value of these coefficients from which the current distributions, phase velocity, and far-field patterns may readily be obtained. Experience has shown that if one chooses a sufficient, number of pointsat which to match boundary conditions, then one can obtain solutions to problems, such as this one, theretofore not easily solvable. In the case of linear elements it has been found that an efficient representation for the current on element n is given by

$$I_n(z') = \sum_{m=1}^{M} I_{nm} \cos\left[(2m-1)\pi z/l_n\right]$$

I_{nm} represent the complex current coefficient of mode m on element n and I_n represents the corresponding length on the n elements. This series of odd-ordered even modes is chosen such that the current goes to zero at the ends of element n. This is a suitable approximation for elements whose diameter is small in terms of the Wavelength.

The theory is based on Pocklington's integral equation for total field generated be an electric current source radiating in an unbounded space as given by the following mathematical analysis.

$$\int_{-1/2}^{1/2} I(z') \; [(\partial^2/\partial z^2) + k^2] \; [(e^{-jkR})/R]dz' = j4\pi\omega\varepsilon_0 E_z^t$$

Where

$R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}$

Since we know that

$(\partial^2/\partial z^2) \; [(e^{-jkR})/R] = (\partial^2/\partial z'^2) \; [(e^{-jkR})/R]$

Putting this into the above equation, we get the reduced form of the Pocklington's integral equation as:

$$\int_{-1/2}^{1/2} I(z') \; [(e^{-jkR})/R]\partial z'^2 + k^2 \int_{-1/2}^{1/2} I(z') \; [(e^{-jkR})/R]dz' = j4\pi\omega\varepsilon_0 E_z^t$$

Now, we will concentrate on the integration of this reduced equation. Integrating the first term by parts where

$$u = I(z')$$

$$du = [dI(z')/dz']dz'$$

$$dv = (\delta^2/\delta z'^2) \; [(e^{-jkR})/R]\partial z' = (\delta/\partial z') \; [(\delta/\partial z') \; (e^{-jkR})/R] \; dz'$$

$$v = (\delta/\partial z') \; (e^{-jkR})/R$$

Reduce it further to

$$\int_{-1/2}^{1/2} (\delta^2/\delta z'^2) \; [(e^{-jkR})/R]\partial z' = I(z') \; [(\delta/\partial z') \; (e^{-jkR})/R]|_{-1/2}^{1/2} - \int_{-1/2}^{1/2} [(\delta/\partial z') \; (e^{-jkR})/R](dI(z')/dz'$$

Since we require that the current at the ends of each wire vanish i.e. I z(z'= +l/2) = Iz (z'= -l/2) = 0, reduces above equation to

$$\int_{-1/2}^{1/2} (\delta^2/\delta z'^2) \; [(e^{-jkR})/R]dz' = \int_{-1/2}^{1/2} (\delta/\partial z') \; [(e^{-jkR})/R]dz'(dI(z')/dz'$$

Integrating by parts where

$$u = dI(z')/dz'$$

$$du = [d^2I(z')/dz'^2]dz'$$

$$dv = (\delta/\delta z') \; [(e^{-jkR})/R]dz'$$

$$v = (e^{-jkR})/R$$

Reduce it to

$$\int_{-1/2}^{1/2} (\delta^2/\delta z'^2) \; [(e^{-jkR})/R]dz' = [dI(z')/dz'] \; [(e^{-jkR})/R] \; |_{-1/2}^{1/2} + \int_{-1/2}^{1/2} (d^2/dz'^2) \; [(e^{-jkR})/R]dz'$$

When this is substituted for the first term, it is firther reduced to

$$[dI(z')/dz'] \; [(e^{-jkR})/R] \; |_{-1/2}^{1/2} + \int_{-1/2}^{1/2} [k^2I(z') + d^2I(z')/dz'] \; [(e^{-jkR})/R]dz' = j4\pi\omega\varepsilon_0 E_z^t$$

For small diameter wires the current on each element can be approximated by a finite series of odd-ordered even modes. Thus, the current on nth element can be written as a Fourier series expansion of following form

$$I_n(z') = \sum_{m=1}^{M} I_{nm} \cos [(2m-1)\pi z/l_n]$$

Where I_{nm} represents the complex current coefficient of mode m on element n and I_n represents the corresponding length of the n element. Taking the Ist and IInd derivatives of above equation and substituting them, results in

$$\sum_{m=1}^{M} I_{nm} \{[(2m-1)\pi/I_n] \sin[(2m-1)\pi z'_n/l_n] \; [(e^{-jkR})/R] \; |_{-ln/2}^{ln/2} + [k^2 - ((2m-1)^2\pi^2/l_n^2) \; . \int_{-ln/2}^{ln/2} \cos[(2m-1)\pi z'_n/l_n]$$

$$((e^{-jkR})/R)dz'_n\} = j4\pi\omega\varepsilon_0 E_z^t$$

Since the cosine is an even function, above equation can be reduced by integrating over only 0<=z'<=l/2 to

$$\sum_{m=1}^{M} I_{nm} \{(-1)^{m+1} [(2m-1)\pi/l_n]\, G_2(x,x',y,y'/z,\, l_n) + [k^2 - ((2m-1)^2\pi^2/l_n^2) \,.\, \int_0^{ln/2} G_2(x,x',y,y'/z,z'_n)$$

$$\cos[(2m-1)\pi z'_n/l_n]dz'_n\} = = j4\pi\omega\varepsilon_0 E_z^t$$

Where

$G_2(x,x',y,y'/z,z'_n) = (e^{-jkR_-})/R\_) + (e^{-jkR_+})/R_+)$

$R_\pm = [(x-x')^2 + (y-y')^2 + a^2 + (z\pm z')^2]^{1/2}$

N = 1,2,3…,N

N = Total number of elements

Where

$R_\pm$ is the distance from the center of the each wire radius to center of any other wire.

The far-field pattern is given by

$$E\theta = \sum_{n=1}^{N} E\theta_n = -j\omega A\theta$$

Where

$$A\theta = \sum_{n=1}^{N} A\,\theta_n = (\mu e^{-jkr}/4\Pi r)\sin\theta \sum_{n=1}^{N} \{ e^{jk(Xn \sin\theta \cos\phi + Yn \sin\theta \sin\phi)} \sum_{m=1}^{M} I_{nm}[\sin(z_+)/(z_+) + \sin(z_-)/(z_-)\ ]\}l_n/2$$

In the Matlab implementation, SINTEG function is for integration. Since integration is very difficult here, so we have used weighted method i.e. Gaussian method which states "In numerical analysis, a quadrature rule is an approximation of the definite integral of a function, usually stated as a weighted sum of function values at specified points within the domain of integration.

V. FLOW CHART

The main concept of the code is based on Pocklington's Integral Equation and is shown below:

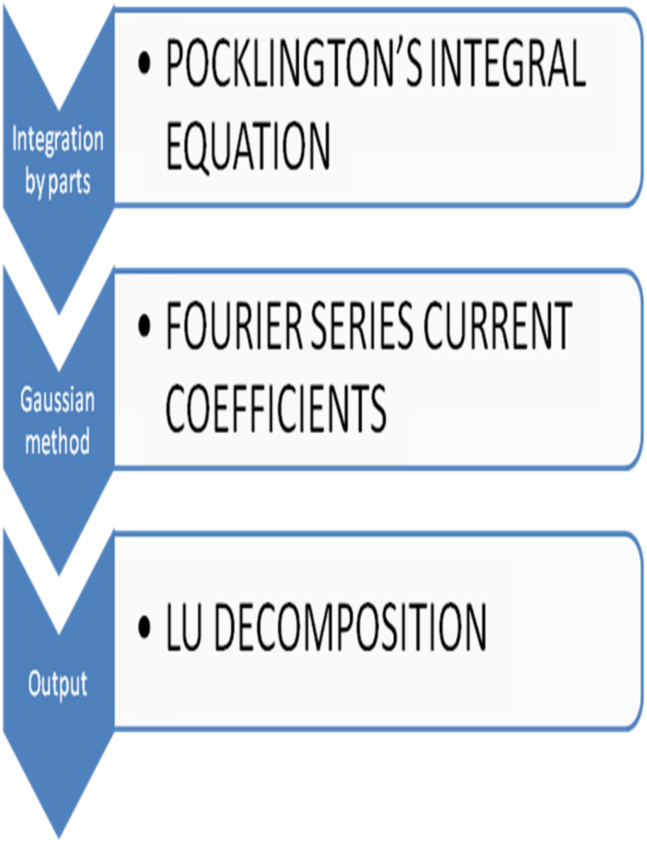

Figure 5.1 Basic Flow Chart

VI. NBS DESIGN

A government document has been published which provides extensive data of experimental investigations carried out by National Bureau of Standards (NBS). We can obtain desired data from the government document.

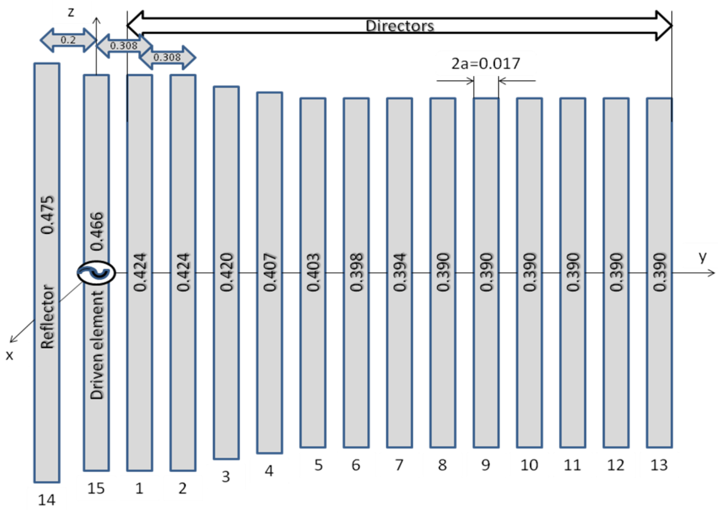

Figure 6.1 NBS Parameters for Yagi-Uda Antenna

- Number of elements , N = 15
- Radius of each element, a = 0.0085
- Director length, $l_1 = l_2 = 0.424$, $l_3 = 0.420$, $l_4 = 0.407$, $l_5 = 0.403$, $l_6 = 0.398$, $l_7 = 0.394$, l_8 to $l_{13} = 0.390$
- Reflector length, $l_{14} = 0.475$
- Feeder (Driven element) length, $l_{15} = 0.466$
- Spacing between directors = 0.308
- Spacing between feeder & reflector = 0.2
- The overall antenna length would be L = 4.2

The parameters (element lengths and spacing) are given in terms of wavelength.

The characteristic variables of the designed NBS antenna can be calculated and listed in the following table:

Directivity	14.2106
Front to back ratio of E-plane	12.0779
Front to back ratio of H-plane	12.0811
3dB beamwidth of E-plane	28.8770
3dB beamwidth of H-plane	30.5268

Table 6.1 Measured Parameters

VII. OPTIMIZATION TECHNIQUES

There are so many ways to optimize the directivity and other antenna parameters. First let's take a look at the most primitive method Trial and Error method.

Trial And Error Method

L_1 = Length of reflector (159cm) = 0.4998λ

L_2 = Length of driven element (149cm) = 0.4684 λ

L_3 = Length of director (140cm) = 0.44.1 λ

S_1 = Spacing between reflector and driven element (44cm) =0.1383 λ

S_2 = Spacing between director and driven element (76cm) = 0.2389 λ

S_3 = Spacing between directors (56cm) = 0.1760 λ

Radius = 0.7cm = 0.0022 λ

Frequency of operation = 94.3MHz

L1	L2	L3	S1	S2	S3	R
150 = 0.4751	140 = 0.4401	130 = 0.4086	35 = 0.1100	65 = 0.2043	45 = 0.1414	0.4 = 0.0013
155 = 0.4872	145 = 0.4458	135 = 0.4244	40 = 0.1257	70 = 0.2200	50 = 0.1572	0.5 = 0.0016
159 = 0.4998	149 = 0.4684	140 = 0.4401	44 = 0.1383	76 = 0.2389	56 = 0.1760	0.7 = 0.0022
165 = 0.5186	155 = 0.4872	145 = 0.4558	50 = 0.1572	80 = 0.2515	60 = 0.1886	1.0 = 0.0031
170 = 0.5344	160 = 0.5029	150 = 0.4715	55 = 0.1728	85 = 0.2672	65 = 0.2043	1.3 = 0.0041

Table 7.1: Conversions

Now varying each parameter sequentially:

For M = 3 & N = 3

L_1		3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
0.4751	L_2 = 0.4684	56.12	69.81	6.7840	6.7794	8.522
0.4872	L_3 = 0.4401	57.63	72.81	12.2094	12.2026	9.293
0.4998	S_1 = 0.1383	58.74	75.17	12.5439	12.5381	9.160
0.5186	S_2 = 0.2389	59.82	77.60	9.7992	9.7952	8.690
0.5344	R = 0.0022	60.42	78.99	8.2683	8.2648	8.352

Table 7.2: Varying Length of Reflector

	L_2	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.4401	59.29	76.40	12.5081	12.5026	9.125
L_3=0.4401	0.4458	59.18	76.16	12.5187	12.5132	9.133
S_1=0.1383	0.4684	58.74	75.17	12.5439	12.5381	9.160
S_2=0.2389	0.4872	58.35	74.32	12.5422	12.5360	9.182
R=0.0022	0.5029	58.01	73.59	12.5231	12.5167	9.200

Table 7.3: Varying Length of Driven Element

	L_3	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.4086	63.69	87.30	17.0513	17.0461	8.047
L_2=0.4401	0.4244	61.77	82.26	16.3498	16.3436	8.538
S_1=0.1383	0.4401	58.74	75.17	12.5439	12.5381	9.160
S_2=0.2389	0.4558	54.09	65.74	7.5843	7.5791	9.521
R=0.0022	0.4715	48.67	56.38	3.0448	3.0400	8.089

Table 7.4: Varying Length of Director

	S_1	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.1100	58.99	75.88	13.5521	13.5459	9.103
L_2=0.4401	0.1257	58.83	75.45	12.9628	12.9569	9.139
L_3=0.4401	0.1383	58.74	75.17	12.5439	12.5381	9.160
S_2=0.2389	0.1572	58.65	74.89	11.9809	11.9752	9.178
R=0.0022	0.1728	58.62	74.76	11.5611	11.5555	9.178

Table 7.5: Varying Spacing Between Reflector And Driven Element

	S_2	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.2043	60.39	79.16	16.7544	16.7475	8.783
L_2=0.4401	0.2200	59.60	77.21	14.5425	14.5362	8.978
L_3=0.4401	0.2389	58.74	75.17	12.5439	12.5381	9.160
S_1=0.1383	0.2515	58.23	74.01	11.4830	11.4773	9.246
R=0.0022	0.2672	57.68	72.74	10.3817	10.3763	9.308

Table 7.6: Varying Spacing Between Director And Driven Element

	R	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.0013	60.34	78.81	14.6641	14.6579	8.864
L_2=0.4401	0.0016	59.77	77.47	13.8845	13.8785	8.978
L_3=0.4401	0.0022	58.74	75.17	12.5439	12.5381	9.160
S_1=0.1383	0.0031	57.41	72.32	10.9441	10.9385	9.351
S_2=0.2389	0.0041	56.12	69.67	9.5508	9.5454	9.476

Table 7.7: Varying Radius of Each Element

For M = 3 & N = 12

	S_3	3dB beamwidth E-plane (Degree)	3dB beamwidth H-plane (Degree)	Front-to-back ratio E-plane (dB)	Front-to-back ratio H-plane (dB)	Directivity (dB)
L_1=0.4998	0.1414	33.47	36.10	5.8692	5.8591	8.483
L_2=0.4401	0.1572	30.75	32.45	8.6325	8.6293	8.609
L_3=0.4401	0.1760	32.45	34.81	16.9353	16.9205	9.993
S_1=0.1383	0.1886	29.94	31.60	8.3003	8.2872	9.829
S_2=0.2389	0.2043	29.19	30.66	9.1289	9.1181	10.801
R=0.0022						

Table 7.8: Varying Spacing Between Directors

The above results show the variation of antenna parameters on changing the element measures.Genetic Algorithm Based Automated Antenna Optimization System: Yagi-uda antennas are known to be difficult to design and optimize due to their sensitivity at high gain and the inclusion of numerous parasitic elements. A genetic algorithm based automated antenna optimization system that uses a fixed Yagi-uda antenna topology and a byte encoded antenna representation, is presented here. The fitness calculation allows the implicit relationship between power gain and sidelobe/backlobe loss to emerge naturally, a technique that is less complex than previous approaches. The genetic operator used is also simple. The result include Yagi-Uda antenna that have excellent bandwidth and gain properties with very good impedance characteristics. Results exceeded previous Yagi-Uda antenna produced via evolution algorithms by at least 7.8% in mainlobe gain.

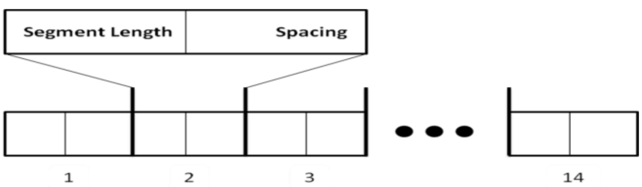

Figure 7.1: Genetic Algorithm Based Antenna

This scheme comprises of 14 elements, each one encoding a length and spacing value. Each floating point value was encoded as three bytes, yielding resolution of 1/224 per value. The first pair of values encoded the reflector element, the second pair encoded the driven element and the remaining 12 pairs encoded the directors. Our point crossover was used with cut points allowed between bytes. Mutation was applied on the individual bytes. Radius values were constrained to 2, 3, 4 or 6mm. All the elements within given individual were assigned the same radius value. Element lengths were constrained to be symmetrical around the x-axis and between 0 and 1.5λ. Elements having zero length were removed from the antenna; as a consequence, a constructed antenna could have less than 14 elements. Spacing between adjacent elements (along the z-axis) was constrained to be between 0.05λ and 0.75λ. The wavelength λ was1.195 and frequency of 235MHz.

VIII. CONCLUSION

The properties of a receiver mode Yagi are relatively uncritical. The bandwidth and VSWR performance matters less than the gain of the antenna and its discrimination against unwanted signals. However, for a transmit Yagi such as is commonly used by Hams and short-wave broadcasters, the accepted power depends critically on getting a good match to the feed. This will vary across the band and is susceptible to the variations in the local environment and geometry distortions.The lore of the Yagi designer has it that the gain of a yagi is governed more by the overall boom length than by the number of elements. For an HF Yagi, the boom length can be a critical factor, and the Ham is usually seeking to optimize the forward gain, the front-to-back ratio and the construction techniques required. Yagi si having thick rod elements (in terms of a wavelength) is better-behaved than those made from thin wires.

The gain of a Yagi-Uda is only moderate, but for the frequency range given above it is cheap and relatively simple to build. It is reasonably tolerant to the variations in construction and indeed many Yagi-Uda designs have been arrived at the cut and try empirical methods.

This is why antenna design is often seen as a black art. With proper numerical simulation, useful improvements have been made to the empirical design. Tradeoffs may be made between the various factors such as, bandwidth, impedance, front-to-back ratio, gain, sidelobe performance and ease of mounting. A vertically polarized Yagi-Uda often is mounted on the top of a vertical conducting mast which, being in the near field and also polarized matched, will modify the electrical properties. There is less of a problem with mounting a horizontally polarized Yagi-Uda antenna.

For moderately long Yagis with several directors, the reflector spacing and size has a little effect on the forward gain, providing that there is a reflector, but being close to the driving element it has a strong effect on the front-to-back ratio and on the driving point impedance of the antenna. The driving element has of course a big effect on the impedance of the structure and it can be tuned to make this impedance nearly real. The directors form the majority of the travelling wave structure.The gain of a Yagi antenna is governed mainly by the number of elements in the particular RF antenna. However the spacing between the elements also has an effect. As the overall performance of the RF antenna has so many inter-related variables, many early design were not able to realize their full performance. Today computer programs are used to optimize RF antenna design before they are manufactured and as a result the performance of antenna has improved.

Number Of Elements	Approximate Anticipated Gain (dB Over Dipole)
2	5
3	7.5
4	8.5
5	9.5
6	10.5
7	11.5

Table 8.1: Parameters

The front-to-back ratio is important in circumstances where interference or coverage in the reverse direction needs to be minimized. Unfortunately the conditions within the antenna mean that optimization has to be undertaken for either.Front-to-back ratio or the maximum forward gain, conditions for both features do not coincide, but the front-to-back ratio can normally be maximized for a small degradation of the forward gain.

REFERENCES

[1] Antenna Theory , Analysis And Design By Constantine A. Balanis
[2] Antenna Engineering Handbook (A.W. Love And T.S. Bird)
[3] R.E. Collin, "Antennas And Radio Wave Propagation"
[4] Robert S. Elliot, "Antenna Theory And Design"
[5] Antenna Theory And Design By Stutzman Thiele
[6] Modern Antenna Design By Thomas A. Milligan
[7] Antenna And EM Modeling With Matlab By Sergey N. Makarov
[8] Antenna Design And Visualization Using Matlab By Atef Z. Elsherbeni And Matthew J. Inman
[9] IEEE Antennas And Propagation Magazine, Vol. 46, No. 5, October 2004.
[10] Antennas And Propagation, IEEE Transactions On [Legacy, Pre - 1988] Volume 27, Issue 2, Mar 1979 Page(S):267 – 270