

## Correlated Emission Laser, Quenching Of Spontaneous Noise and Coupled Pendulum Analogy

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**Abstract:** The laser is an acronym for Light Amplification by Stimulated Emission of Radiation. This is a unique beam of light which has properties of spatial coherence and temporal coherence simultaneously. The fundamental source of noise in a laser is spontaneous emission. This is also related to the so called laser linewidth which is attributed to the random phase diffusion process arising from the addition of spontaneously emitted photons with random phases to the laser field. In the present work we show that the quantum noise leading to the laser linewidth can be suppressed below the standard limit (Schalow-Townes limit) by preparing the atomic systems in a coherent superposition of states as in the Hanle -Effect and Quantum Beat experiments. In such coherently prepared atoms the spontaneous emission is said to be correlated. Lasers operating via such a phase coherent atomic ensemble are known as correlated spontaneous lasers (CEL). An interesting aspect of Correlated Emission Laser is that it is possible to eliminate the spontaneous quantum noise in the relative linewidths by correlating the two spontaneous emission noise events. A well defines coherence between the upper and lower levels leads to a correlation between the light emitted by a transition in a three level system. In this work we present a coupled pendulum analogy which illustrates CEL.

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### I. Introduction:

At the very outset we would like to emphasize that spontaneous emission is a universal phenomenon. The origin of the spontaneous emission can be understood only with the help of Weisskopf – Wigner theory [1] which implies that the spontaneously emitted radiation is not perfectly monochromatic but instead has a frequency spectrum with width inversely proportional to the lifetime  $\tau$ . The mechanism of spontaneous emission can be understood from the quantum theory of radiation [2]. It is an isotropic perturbation always present and attributed in connection with the quantum theory of radiation to the all pervading zero – point fluctuation of the electromagnetic field. The light excites the atoms : the zero – point fluctuations deexcite them resulting in re-radiation of light. It may be noted that zero- point energy or vacuum fluctuation is the consequence of the quantization of the radiation field . With reference to laser ( or stimulated emission ) it is worthwhile to say that spontaneous emission is always present in the lasing wavelength as noise, even as the wheel follows the hoof of the draught – ox, or like ones shadow that never leaves. In other words petrol or diesel drives an engine and similarly the spontaneous emission drives the stimulated emission. Now the question arises whether it is possible to remove the spontaneous noise so as to observe the pure spectral line free from spontaneous noise? The answer is not easy. But yes. A simple pictorial model for the origin of the laser linewidth envisions it as being due to the random phase diffusion process arising from the addition of spontaneously emitted photons with random phases to the laser field. The quantum noise leading to the ,laser linewidth can be suppressed below the standard Schalow – Townes limit by preparing the atomic systems in a coherent superposition of states as in the Hanle effect [3] and quantum beat experiments [4 -7]. In such coherently prepared atoms the spontaneous emission is said to be correlated. Lasers operating via such a phase coherent atomic ensemble are known a correlated emission lasers (CEL). An interesting aspect of the correlated emission lasers is that it is possible to eliminate the spontaneous emission quantum noise in the linewidths by correlating the two spontaneous emission noise events. In the present work we report a coupled pendulum analogy for correlated emission laser (CEL) and quenching of spontaneous emission.

## II. Theory and concept:

In active laser interferometer experiments, the limiting source of quantum noise is often spontaneous emission fluctuations in the relative phase angle. The diffusion of the relative phase angle between two such laser modes may be eliminated by preparing a laser medium consisting of three level atoms and arranging that the two transitions  $|a\rangle \rightarrow |c\rangle$  and  $|b\rangle \rightarrow |c\rangle$  drive a doubly resonant cavity as shown as in Fig 1 (a). In this way the optical paths may be differently affected by an external influence. [e.g. gravity wave].

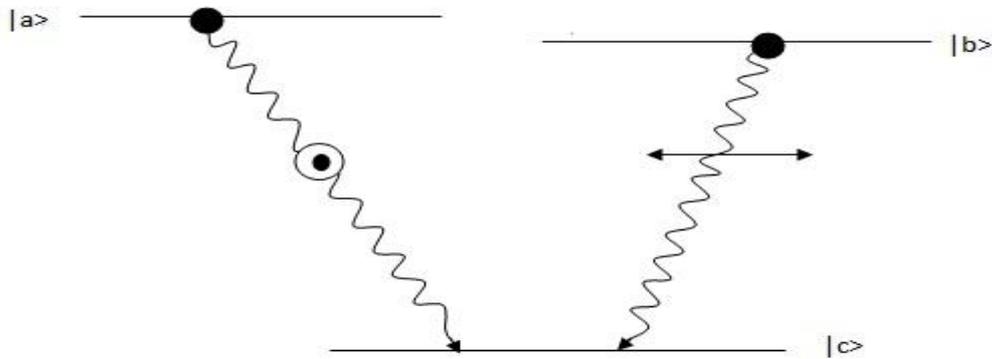
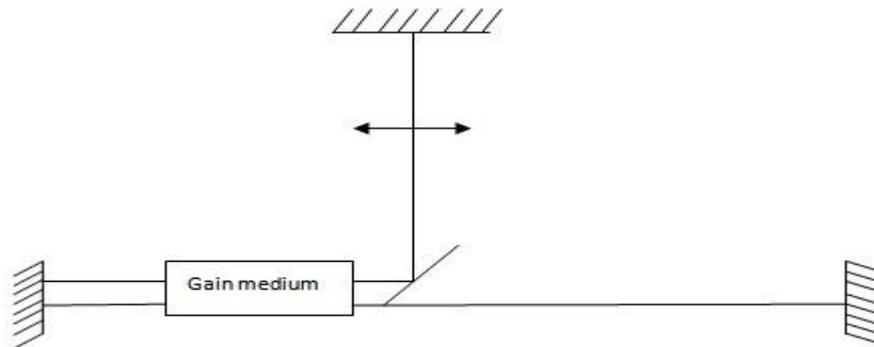


Fig. 1(a)

Hanle laser (a) The atoms are coherently pumped into the lasing levels  $|a\rangle$  and  $|b\rangle$  through an appropriately polarized pump beam. Transition from these states to the common ground state  $|c\rangle$  differ in their polarization. (b) Scheme of the Hanle laser using the coherently pumped atoms as active medium. A polarization sensitive mirror separates the polarization modes in the doubly resonating cavity.



(b) Doubly Resonant Cavity.

The atomic transitions driving the two optical paths are strongly correlated when the upper levels  $|a\rangle$  and  $|b\rangle$  are prepared in a coherently superposition as in Hanle effect or quantum beat experiments. In the Hanle effect example, the levels  $|a\rangle$  and  $|b\rangle$  can be taken to be 'linear polarization' states formed from a single elliptical polarization state as shown in Fig 1 (a). In the quantum beat case the coherent mixing is produced by a strong external microwave signal as shown in Fig 2. The fields emitted by the atoms in Fig 1 will differ in polarization while the fields produced by the atoms in Fig ( 2 ) will differ in frequency. In both cases the heterodyne beat notes between the spontaneously emitted fields 1 and 2 shows that they are strongly correlated.

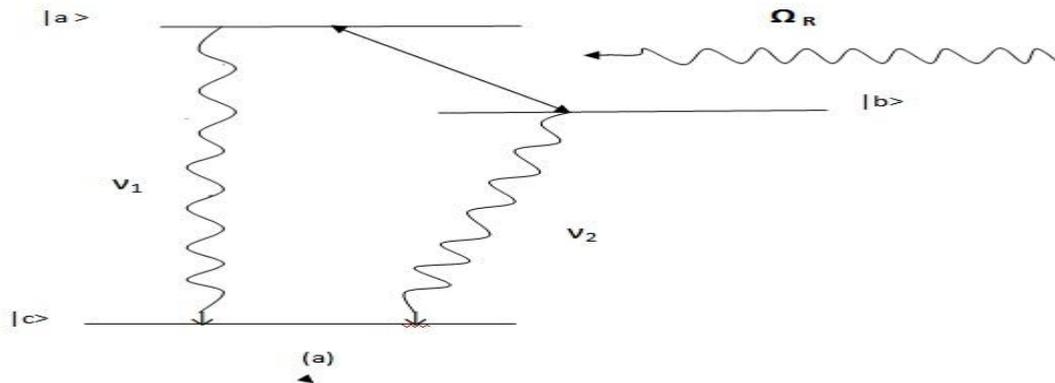


Fig2. The atoms are prepared in a coherent superposition of upper levels  $|a\rangle$  and  $|b\rangle$  by an external field with an effective Rabi frequency  $\Omega_R$ . The two laser transitions at frequencies  $\nu_1$  and  $\nu_2$  share a common lower level  $|c\rangle$ .

(b) Scheme of the quantum beat laser with doubly resonant cavity.

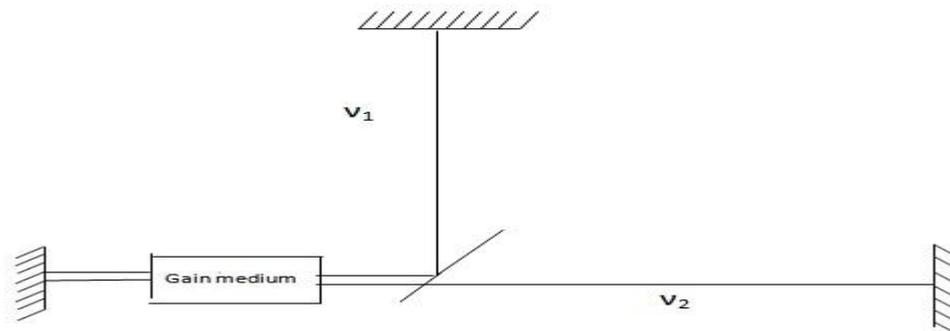


Fig.2 (b) Doubly Resonant Cavity

Without going into the mathematical details we can understand the quenching of the spontaneous emission fluctuations in the relative by phase  $\Theta$  by referring to Fig 3. Here we consider the random walk of the tips of the electric field phases of the two modes in the complex  $\alpha$  - plane. If we ignore amplitude fluctuations, the phase fluctuations associated with the spontaneous emission allow the tips of the fields to diffuse out around a circle in the complex plane. When the so called diffusion co-efficient  $D_{\Theta\Theta} = 0$  the spontaneous emission in the modes become highly correlated so that the relative phase angle is  $\Theta$  locked to a particular value. The average phase variables has, however, non-vanishing diffusion.

Like the Hanle effect a quantum beat laser consists of three level atoms in the V configuration which are pumped in the upper level  $|a\rangle$  inside a doubly resonant cavity. A coherence is introduced between the upper levels  $|a\rangle$  and  $|b\rangle$  by an external field which is characterized by the Rabi frequency  $\Omega_R \exp(-i\phi)$  where  $\Omega_R$  and  $\phi$  are the real amplitude and phase. The transition  $|a\rangle \rightarrow |c\rangle$  and  $|b\rangle \rightarrow |c\rangle$  are assumed dipole allowed. The transition  $|a\rangle \rightarrow |b\rangle$  is therefore dipole forbidden. The external field leading to coherence between the levels could be a strong magnetic field for a magnetic dipole allowed transitions.

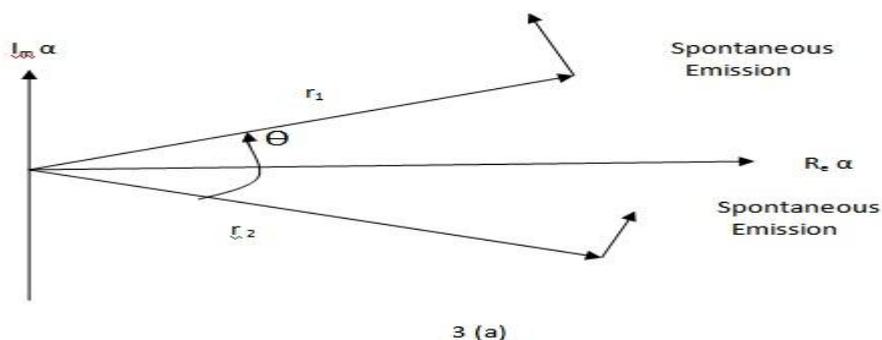


Fig 3 (a) : Pictorial representation of the correlated emission effect. The spontaneous emission events in the two modes are highly correlated and the relative phase remains the same. (a) before and (b) after the spontaneous emission events.

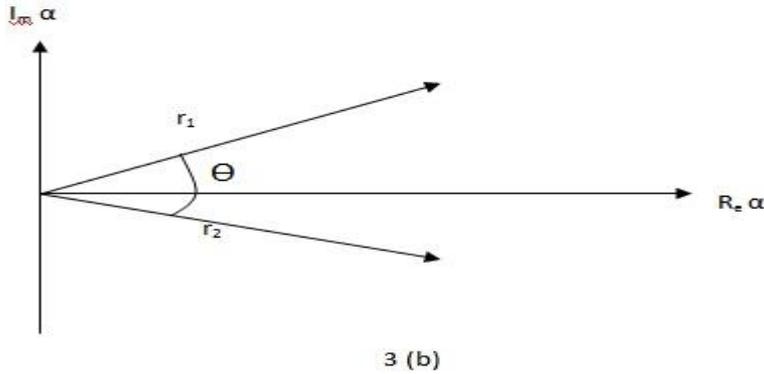


Fig. 3.

representation:

Coupled pendula

From what has been discussed in earlier sections [ mathematical details omitted] one observes that spontaneous emission noise may be eliminated via correlated emission. It is worthwhile to indicate here that there are also other paths available to arrive at the goal. In the present section we provide a simple analogy which illustrates some aspects of CEL . The mechanical analogy is shown in Fig 4. Two pendula are hung from a rigid spring as shown. The lengths of the pendula are such that they differ considerably, as for example length of one is twice that of the other.

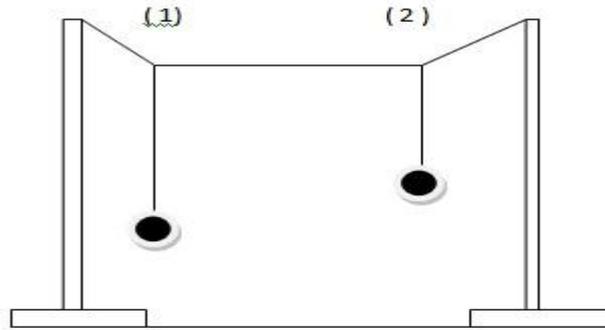


Fig. 4: Coupled Pendulum analogy of CEL

If only pendulum (1) is given initial amplitude and released, it will generate vibration of the second pendulum (2) until pendulum (2) reaches some maximum amplitude. Following this condition the energy from pendulum (2) will pass gradually back to pendulum (1). Pendulum (1) in the process will not lose all of its energy to pendulum (2). The mechanical analogy can further be extended to a number of pendula coupled to the rigid string. The physical significance of this model is that the amplitude is the exact analogue to the spontaneous noise. This noise or amplitude does not become zero but it is transferred to another pendulum. We say that the amplitudes are quenched through processes which are correlated. In CEL also the spontaneous noises are quenched or to be precise we say that the spontaneous noises have the probability that they will be quenched via CEL. In conclusion, we like to state that complete quenching of spontaneous emission is ideally not possible, but it is possible to achieve this goal for a short period of time. As in the case of a coupled pendulum where the amplitude goes down decreasing but it is increasing again.

Consider the system in Fig 5. If the atoms are prepared in a coherent superposition of  $|a\rangle$  and  $|b\rangle$  then the difference of the corresponding phases  $\phi_a - \phi_b$  is fixed the phase  $\phi_c$  of the ground level  $|c\rangle$  is however, a true random variable. The spontaneously emitted fields in the  $|a\rangle - |c\rangle$  and  $|b\rangle - |c\rangle$  therefore average to zero. But from the beat signal of the two spontaneously emitted fields, the random phase  $\phi_c$  cancels. This is the idea of the noise contribution first introduced by Bergou et al. [8]. The physical condition under which it occurs is that the field in the detuning from the corresponding atomic lines are equal to the half of the Rabi frequency of

the driving field that coherently mixes the upper levels and they are much larger than the atomic decay constants.

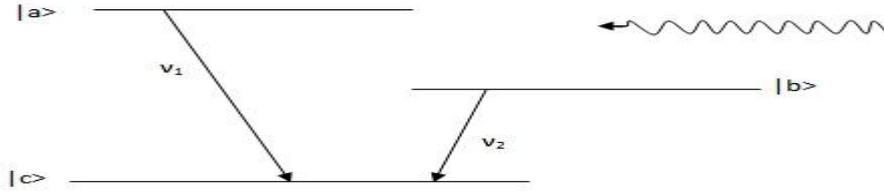


Fig 5. In the three level atoms, the two upper levels  $|a\rangle$  and  $|b\rangle$  are coupled via a strong microwave frequency  $\nu_3$ . The emissions from the  $(|a\rangle \rightarrow |c\rangle)$  and  $(|b\rangle \rightarrow |c\rangle)$  transitions are strongly correlated.

It is worthwhile to write the master equation equivalent to a Fokker – Planck equation for the P-representation in the following form (9)

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \theta} (d_\theta P) - \frac{\partial}{\partial \mu} (d_\mu P) + \frac{\partial^2}{\partial \theta^2} (D_{\theta\theta} P) + \frac{\partial^2}{\partial \mu^2} (D_{\mu\mu} P) \dots \dots \dots (1)$$

The physical meaning of the terms in Equation (1) is the following. The co-efficients  $d_\theta$  and  $d_\mu$  are the drift coefficients with respect to the variables

$\theta = \theta_1 - \theta_2$  and  $\mu = (\theta_1 + \theta_2)/2$ .  $D_{\theta\theta}$  and  $D_{\mu\mu}$  are the corresponding diffusion co-efficients. The characteristics feature of the diffusion and drift- co- efficient is that they are phase dependent and these phase dependences arise due to the injected coherence. A much simpler set of co-efficient is obtained when

$$P_{aa}^{(0)} = P_{bb}^{(0)} = |P_{ab}^{(0)}| = |P_{ba}^{(0)}| = \frac{1}{2} \quad (2)$$

$$g_1 = g_2 = g \text{ and } \langle n_1 \rangle = \langle n_2 \rangle = \langle n \rangle$$

Under these conditions

$$d_\theta = -\frac{A}{2} \sin \psi \quad (3)$$

$$d_\mu = -\frac{A\mu}{2} (1 + \cos \psi) \quad (4)$$

$$2 D_{\theta\theta} = \frac{A}{2} (1 - \cos \psi) \quad (5)$$

$$2 D_{\mu\mu} = 0 \quad (6)$$

$$2 D_{\mu\mu} = \frac{A}{2} (1 + \cos \psi)$$

Where  $A = 2 r_a g^2 / (\gamma^2 + \Delta^2)$  is the linear gain co- efficient.

From Fokker-Planck equation (1) one can derive the following equation of motion for the relative

phase

$$\frac{\partial}{\partial t} \langle \theta \rangle = \langle d_\theta \rangle \quad (8)$$

Phase locking [ $\langle \theta \rangle = \text{constant}$ ] takes place for those values of  $\theta$  for which the drift co- efficient vanishes. This happens when  $\psi = 0$  is

$\theta = \phi$ . We also see that the diffusion co- efficient for the relative phase angle (5) which proportional to  $(1 - \cos \psi)$  vanishes when the angle  $\psi$  itself vanishes.

These above mentioned conditions can also be realized in the case of coupled pendula.

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