

## Detection and localization method of Single and Simultaneous faults

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**Abstract** – In this paper, we are interested in monitoring a dynamic system in order to contribute to the establishment of a general procedure for fault detection. The detection mechanism is based on the mathematical model of the process; we use an observer to generate residuals for a decision in a stage of monitoring and diagnostic system when disturbance or defects occur. Our contribution is the proposal of a diagnostic method when multiple and simultaneous faults affect the system. A simulation example is used to demonstrate the performance of the proposed fault diagnosis method.

**Keywords** - Fault detection, Fault location, observer, residues.

### I INTRODUCTION

Fault diagnosis has been becoming more and more important for process monitoring because of the increasing demand for hi safety and reliability of dynamic systems. Fault diagnosis is a major research topic attracting considerable interest from industrial practitioners as well as academic researchers. There exist a lot of research works related with fault detection. Most of the methods used are analytic, based on artificial intelligence (AI) or statistical methods. [1] classifies fault detection and diagnosis methods in three groups: Quantitative Model Based, Qualitative Model Based and Process History Based. (See fig. 1).

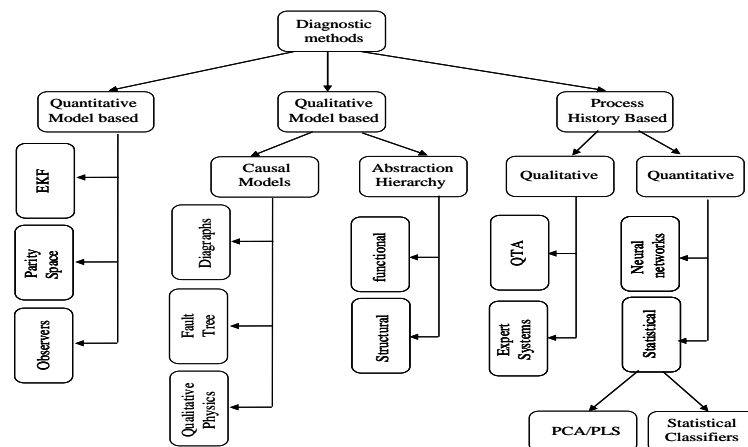


Figure 1: Diagnosis methods

Quantitative Model Based fault detection methods are based on a mathematical model of the system. The occurrence of a fault is captured by discrepancies between the observed behavior and the one that is predicted by the model. These approaches make use of state estimation, parameter identification techniques, and parity relations to generate residuals. However, it is often difficult and time-consuming to develop accurate mathematical models that characterize all the physical phenomena occurring in industrial processes.

Qualitative Model Based fault detection methods use symbolic reasoning which generally combines different kinds of knowledge with graph theory to analyze the relationships between variables of a system. An advantage of these methods is that an explicit model of the system to be diagnosed is not necessary. Knowledge-based approaches such as expert systems may be considered as alternative or complementary approaches where analytical models are not available.

Process History Based fault detection methods only require a big quantity of historical process data. There are several ways in which these data can be transformed and presented as prior knowledge of a system. These transformations are known as feature extraction and could be qualitative, as those used by expert systems, and qualitative trend analysis methods or quantitative, as those used in neural networks, PCA, PLS or statistical pattern recognition.

Very recently, the need to develop more powerful approaches has been recognized, and hybrid techniques that combine several reasoning methods start to be used [2] incorporates model based diagnosis and signal analysis with neural networks. [3] Proposed an approach based on four independent artificial neural networks (ANN) for real time fault detection and classification in power transmission lines. The technique uses consecutive magnitude current and voltage data at one terminal as inputs to the corresponding ANN. The ANN outputs are used to indicate simultaneously the presence and the type of the fault.

Most of the work on observer model-based approaches has been based on using general input-output and state space models to generate residuals. These methods are very effective for the detection and fault location. Indeed, they have grown considerably in the case of linear systems [4], [5] and [6]. In this paper, the mathematical-based approach is adopted to build a complete diagnostic system, able to detect in a simple and easy way simple, multiple, simultaneous and non-simultaneous faults, as well as capable to diminish the false alarms rate. We will focus on simultaneous defaults from the actuators and sensors (fig. 2). At first, a brief review on different methods of detection and fault isolation is presented. We focus on the basic concepts of diagnostic systems based on different models and structures of residual generation.

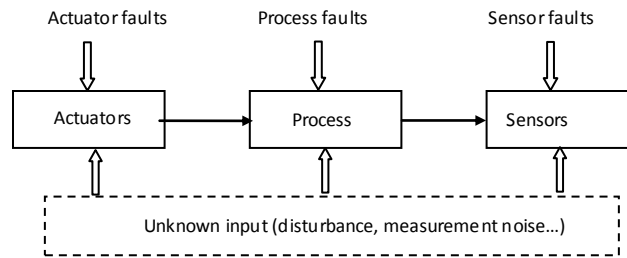


Figure 2: faults of a process

The proposed approach is illustrated in a hydraulic system with three tanks. The last part of this paper will be reserved for simulation results of obtained models.

## II MODEL-BASED DIAGNOSIS

Different approaches for fault-detection using mathematical models have been developed in the last 20 years; see [3], [4], [5], [6], [7] and [8]. The task consists of the detection of faults in the processes, actuators and sensors by using the dependencies between different measurable signals. These dependencies are expressed by mathematical process models. Different types of models are used : a methods for fault-detection using mathematical model are based on measured input and output signals, the detection methods generate residuals, parameter estimates or state estimates (community FDI), which are called features. However, analytical models are used as models of type knowledge base are applied by the community for the (FX) expert systems approach.

By comparison with the normal features (nominal values), changes of features are detected, leading to analytical symptoms. The model used directly as a reference for failure detection, the quality of the result depends directly on the quality of the models. In this work, our goal is to generate several symptoms indicating the difference between nominal and faulty status when a simultaneous defaults from actuators or/and sensors are applied. The principle of the detection model is shown in Figure 3.

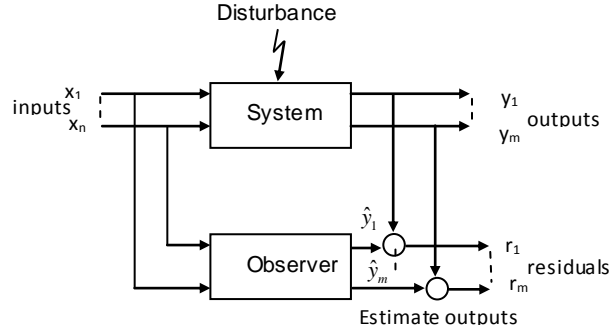


Figure 3: Detection model of defects

The outputs system are  $y_1 \dots y_m$  and the inputs system are  $x_1 \dots x_n$ .

### III THEORETICAL BASIS

There are different schemes based on observers to detect and isolate faults in dynamic processes. Diagnosis schemes based on observers can be classified according the type of fault detected: sensor faults (Instrument Fault Detection or IFD), actuator faults (Actuator Fault Detection or AFD), and component faults (Component Fault Detection or CFD). When several observers constitute a bank of observers of reduced order we have a Dedicated Observer Scheme (DOS). For faults in sensors (IFD), each observer uses all the inputs and just one output. The number of observers equals the number of outputs (sensors). For actuator faults (AFD) each observer uses one input and all the outputs. It should be mentioned that the DOS scheme allows the localization of multiple faults, either in sensors (IFD) or in actuators (AFD). The Generalized Observer Scheme (GOS) is formed by a bank of observers of reduced order. For faults in sensors (IFD), each observer uses all the inputs and  $m-1$  outputs, where  $m$  is the number of outputs. For actuator faults (AFD), each observer uses all the outputs and  $n-1$  inputs,  $n$  being the number of inputs. If the process parameters are known, either state observers or output observers can be applied, [9]. In our case, we suppose that we have a representation of the process as a linear dynamic model with  $n$  inputs, denoted  $x(t)$  and  $m$  measured outputs, denoted  $y(t)$ . The set of  $n$  variables describing the state of the process, denoted  $x(t)$ . In these conditions, the state system can be expressed by the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

$$x(0) = x_0 \quad (3)$$

Where the matrices  $A$ ,  $B$  and  $C$  have compatible dimensions with those of the vectors  $x(t)$ ,  $u(t)$  and  $y(t)$  [10]. In our case we will consider the observatory model can write:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)) \quad (4)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5)$$

$$\hat{x}(0) = \hat{x}_0 \quad (6)$$

Where  $\hat{y}(t)$  et  $\hat{x}(t)$  are estimate outputs and inputs.

$K$  is a matrix such that the term  $(A - KC)$  is stable.

#### 3.1 MULTIPLE FAULT DETECTION

In this work, we assume that the simultaneous actuators and sensors are affected. The residuals generated must be sensible to these faults and detect there. In this case we can isolate the faults. Under these conditions, the system affected by these various defects can be introduced by the following equations [12]

$$\dot{x}(t) = Ax(t) + Bu(t) + Fw(t) + D_a f_a(t) \quad (7)$$

$$y(t) = Cx(t) + D_s f_s(t) \quad (8)$$

$$x(0) = x_0 \quad (9)$$

Where:

$w(t)$  is the unknown input,  $f_a(t)$  is a failure of the actuators,  $f_s(t)$  is a sensor failure.

$F$ ,  $D_a$  and  $D_s$  are matrices of distribution, respectively, of unknown inputs, actuator and sensor. The vectors  $f_a(t) = [f_{a1}(t) \dots f_{ar}(t)]^T$  and  $f_s(t) = [f_{s1}(t) \dots f_{sm}(t)]^T$  assume values different from zero only in the presence of faults.

Usually these signals are described by step and ramp functions representing abrupt and incipient faults (bias or drift), respectively.

### 3.2 ESTIMATION OF UNKNOWN INPUTS

$$\dot{y}(t) = C\dot{x}(t) + D_s \dot{f}_s(t) \quad (10)$$

$$\dot{y}(t) = CAx(t) + CBu(t) + CFw(t) + CD_a f_a(t) + CD_s \dot{f}_s(t) \quad (11)$$

From this relationship we can write:

$$\hat{w}(t) = (CF)^+ (\dot{y}(t) - CA\hat{x}(t) - CBu(t) - CD_a f_a(t) - CD_s \dot{f}_s(t)) \quad (12)$$

Where the term  $CF$  is defined by the following relation [12]:  $CF^+ = ((CF)^T CF^{-1})(CF)^T$  and  $(CF)^+ CF = I_q$

Substituting equation (11) in (12), we can write:

$$\hat{w}(t) = (CF)^+ CA(x(t) - \hat{x}(t)) + w(t) \quad (13)$$

$$\hat{w}(t) = (CF)^+ CA\tilde{x}(t) + w(t) \quad (14)$$

$\tilde{x}(t)$  Will be made to approach zero asymptotically. Then  $\hat{w}(t)$  is an estimation of  $w(t)$ .

### 3.3 ESTIMATION IN THE PRESENCE OF ACTUATOR FAULTS

Consider a system whose actuators may be affected by additive faults  $f_a(t)$ . From equation (4) we can write:

$$\dot{\tilde{x}}(t) = (A - KC)\tilde{x}(t) + D_a f_a(t) \quad (15)$$

$$\tilde{y}(t) = C\tilde{x}(t) \quad (16)$$

$$\tilde{x}(0) = x_0 - \hat{x}_0 \quad (17)$$

The Laplace transform of these equations allows to write:

$$\tilde{Y}(p) = C(pI - A + KC)^{-1} D_a F_a(p) \quad (18)$$

So, the synthesis of several observers, each dedicated to a fault, can efficiently locate each defect [13].

### 3.4 ESTIMATION IN THE PRESENCE OF SENSOR FAULTS

The same reasoning as the previous one, leads us to the following system of equations:

$$\dot{\tilde{x}}(t) = (A - KC)\tilde{x}(t) + KD_s f_s(t) \quad (19)$$

$$\tilde{y}(t) = C\tilde{x}(t) + D_s f_s(t) \quad (20)$$

$$\tilde{x}(0) = x_0 - \hat{x}_0 \quad (21)$$

This gives:

$$\tilde{Y}(p) = (I - C(pI - A + KC)^{-1} K) D_s F_s(p) \quad (22)$$

$$D_s F_s(p) = (C(pI - A)^{-1} K + I) \tilde{Y}(p) \quad (23)$$

Finally if we define the matrix  $D_s^{+1}$  which satisfies the condition  $D_s^{+1}D_s = 1$  the estimate sensor faults is given by the following filter [12]:

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A-KC & 0 \\ -KC & A \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B & K \\ 0 & K \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \quad (24)$$

$$f_s(t) = \begin{bmatrix} -D_s^{+1}C & D_s^{+1}C \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} 0 & D_s^{+1} \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \quad (25)$$

## IV MODELING OF THE EXAMPLE SYSTEM

### 4.1 THE PROCESS PRESENTATION

We use the system depicted in Figure 4 to illustrate our approach. The system consists of three liquid tanks that can be filled with two identical, independent pumps acting on the outer tanks 1 and 2. The tanks are interconnected to each other through upper and lower pipes with resistances  $az_1$  and  $az_2$  to restrict the flow rates  $q_1$  and  $q_2$ . Liquid ( $q_3$ ) can only leave through the outlet pipe below tank 3 and encountering resistance  $az_3$ . The heights  $h_1, h_2$  and  $h_3$  of the tanks are taken as both state variables and observed variables as well.

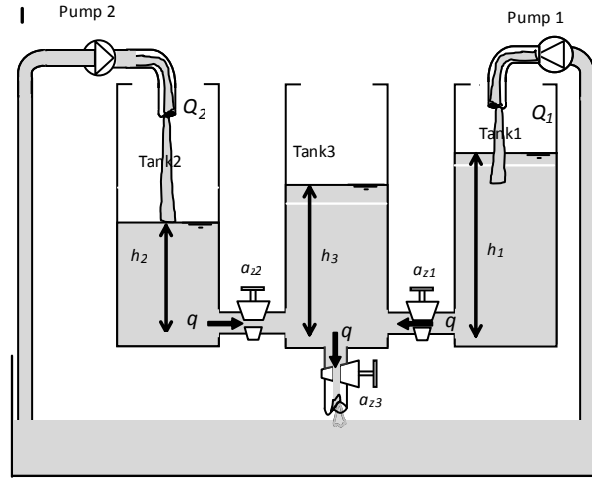


Figure 4: The process of tree tanks

### 4.2 SYSTEM MODELLING

The model can be represented in the form of equation (1) and (2) with  $\dot{h}_i = \frac{d}{dt} h_i$ . It is a control system with 2 inputs and 3 outputs.

$$\dot{h}_i(t) = \frac{1}{A_i} (Q_i^{in}(t) - Q_{ij}^{out1}(t) - Q_{ij}^{out2}(t)) \quad i, j = 1, 2, 3 \quad (26)$$

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{pmatrix} = \begin{bmatrix} -c_1 \text{sign}(h_1 - h_3) \sqrt{|h_1 - h_3|} - B_1 \sqrt{h_1} + \frac{Q_1}{A_1} \\ c_3 \text{sign}(h_3 - h_2) \sqrt{|h_3 - h_2|} - (B_4 + B_2) \sqrt{h_2} + \frac{Q_2}{A_2} \\ \frac{dh_3}{dt} = c_1 \text{sign}(h_1 - h_3) \sqrt{|h_1 - h_3|} - (B_3 + B_2) \sqrt{h_3} - c_3 \text{sign}(h_3 - h_2) \sqrt{|h_3 - h_2|} \end{bmatrix} \quad (27)$$

Where  $A_i$  is the cross-section area of the tanks.

$C_\alpha$  and  $B_\beta$  coefficients given respectively by:

$$c_i = \frac{1}{A} a_{zi} S_n \sqrt{2g} \quad i = 1, 3 \quad (28)$$

$$B_j = \frac{1}{A} b_{zj} S_L \sqrt{2g} \quad j = 1, 2, 3, 4 \quad (29)$$

If  $q_{\text{ext}}$  take  $B_1 = B_2 = B_3 = 0$ , the system becomes three equations:

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{pmatrix} = \begin{bmatrix} -c_1 \text{sign}(h_1 - h_3) \sqrt{|h_1 - h_3|} + \frac{Q_1}{A_1} \\ c_3 \text{sign}(h_3 - h_2) \sqrt{|h_3 - h_2|} + B_4 \sqrt{h_2} + \frac{Q_2}{A_2} \\ c_1 \text{sign}(h_1 - h_3) \sqrt{|h_1 - h_3|} - c_3 \text{sign}(h_3 - h_2) \sqrt{|h_3 - h_2|} \end{bmatrix} \quad (30)$$

The input variable matrix U is set to be:

$$U^T = [u_1 \ u_2]^T = [Q_1 \ Q_2]^T \quad (31)$$

As mentioned above, both the state variable and the observed output are the liquid levels in the three tanks.

$$X^T = [x_1 \ x_2 \ x_3]^T = [h_1 \ h_2 \ h_3]^T \quad (32)$$

$$Y^T = [y_1 \ y_2 \ y_3]^T = [h_1 \ h_2 \ h_3]^T \quad (33)$$

The matrices  $A$ ,  $B$  and  $C$  are:

$$A = \begin{bmatrix} -\frac{1}{A_1 a z_1} & \frac{1}{A_1 a z_1} & 0 \\ 0 & \frac{1}{A_2 a z_2} & -\frac{1}{A_2 a z_2} \\ \frac{1}{A_2 a z_1} & -\frac{1}{A_2} \left( \frac{1}{a z_1} + \frac{1}{a z_2} + \frac{1}{a z_3} \right) & \frac{1}{A_2 a z_3} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

At equilibrium water levels in the three columns are constant, while its derivatives are zero.

$$\dot{h}_1 = \dot{h}_2 = \dot{h}_3 = 0 \quad (34)$$

$c_1 \text{sign}(h_1 - h_3) \geq 0$  and  $c_3 \text{sign}(h_3 - h_2) \geq 0$ , Then

$$\text{sign}(h_1 - h_3) = \text{sign}(h_3 - h_2) = 1 \quad (35)$$

We consider:  $x_1 = h_1$ ,  $x_2 = h_2$ ,  $x_3 = h_3$ ,  $u_1 = Q_1$  and  $u_2 = Q_2$

Our system can be written as follows:

$$\begin{cases} \dot{x}_1 = -c_1 \sqrt{|x_1 - x_3|} + \frac{u_1}{A_1} \\ \dot{x}_2 = c_3 \sqrt{|x_3 - x_2|} - B_4 \sqrt{x_2} + \frac{u_2}{A_2} \\ \dot{x}_3 = c_1 \sqrt{|x_1 - x_3|} - c_3 \sqrt{|x_3 - x_2|} \end{cases} \quad (36)$$

$$\begin{cases} \dot{x} = f(t, x) + gu \\ y = cx \end{cases} \quad (37)$$

### 4.3 MODEL IN SIMULINK

Simulink tool is very powerful platform in system simulation and control. It has the advantages of object and convenient in use. Therefore, it has been widely applied in many control systems simulation and design . Figure 5 shows the Simulink model of the three-tank system.

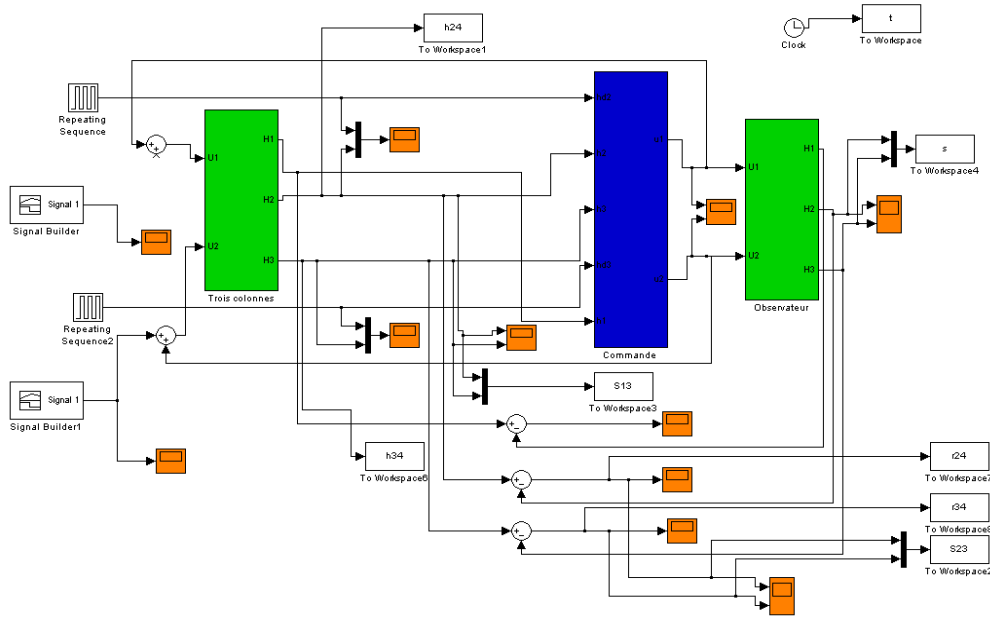


Figure 5: simulink model of a three tank system

The inputs are chosen to unit step signal with specific gains, which can be directly obtained in the Source Library in Simulink.

The parameters in the model are listed in Table 1.

**Table1: The Structural Parameters Of The Three-Tank System**

Parameters	$A_1 (m^2)$	$A_2 (m^2)$	$A_3 (m^2)$
Values	0.0154	0.0154	0.0154
Parameters	$a_{z1}(sec/m^2)$	$a_{z2}(sec/m^2)$	$a_{z3}(sec/m^2)$
Values	$1.75 \cdot 10^4$	$1.15 \cdot 10^4$	$2.25 \cdot 10^4$

## V RESULTAT OF SIMULATION

Four different scenarios will be considered. In the first case, it is assumed that the output faults measuring the levels  $h_2$  and  $h_3$  and generated residuals (Figure 6). In the second case, fault is injected in the first input which is representing the actuator U1, (Figure 7) and in the input which is representing the actuator U2, (Figure 8). The measured value is suddenly deviated from the normal measurement, respectively after 5 seconds for U1 and 4 seconds for U2. In the Third case, we consider that fault is occurred in the sensors measuring  $h_2$  or  $h_3$  (Figure 9 and figure10). Fault has been occurred after 6 seconds in defect sensor. In our case is  $h_2$ .

In the last case, we considered the case when the actuators and the sensors are defects simultaneously (Fig. 11). In this case the observer can isolate the element of the component in default.

All cases have been simulated and residuals have been achieved. As it can be seen in these figures, successful fault detection has been achieved. However, this approach is successful in fault detection and isolation.

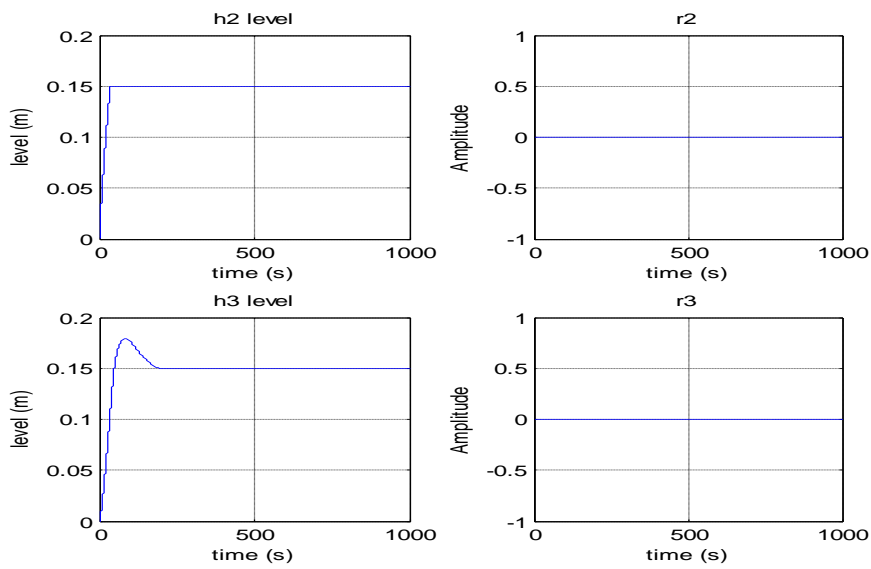


Figure 6: Levels and residuals of the two tanks in the absence of the faults

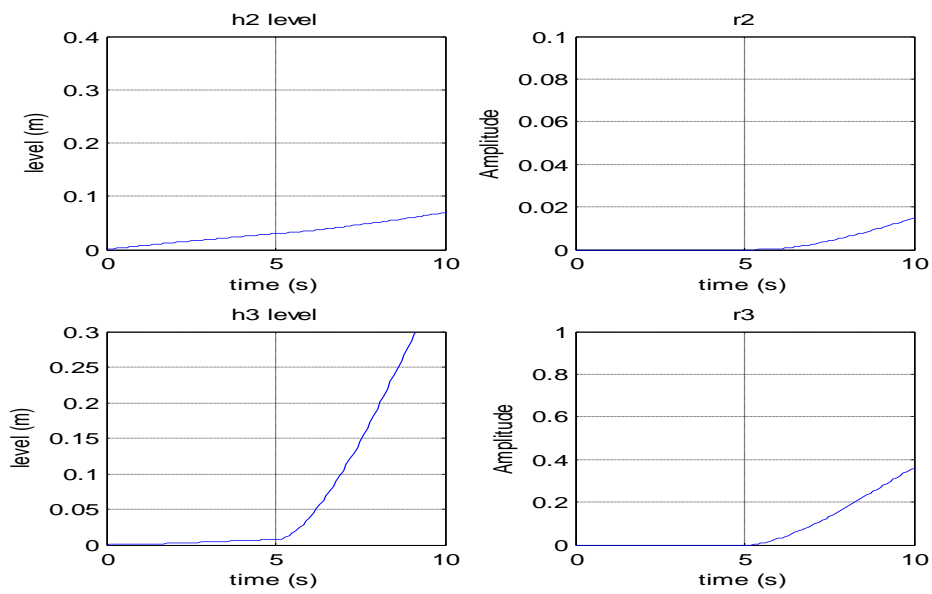


Figure 7: Output system and residuals in the presence of a fault in actuator U1



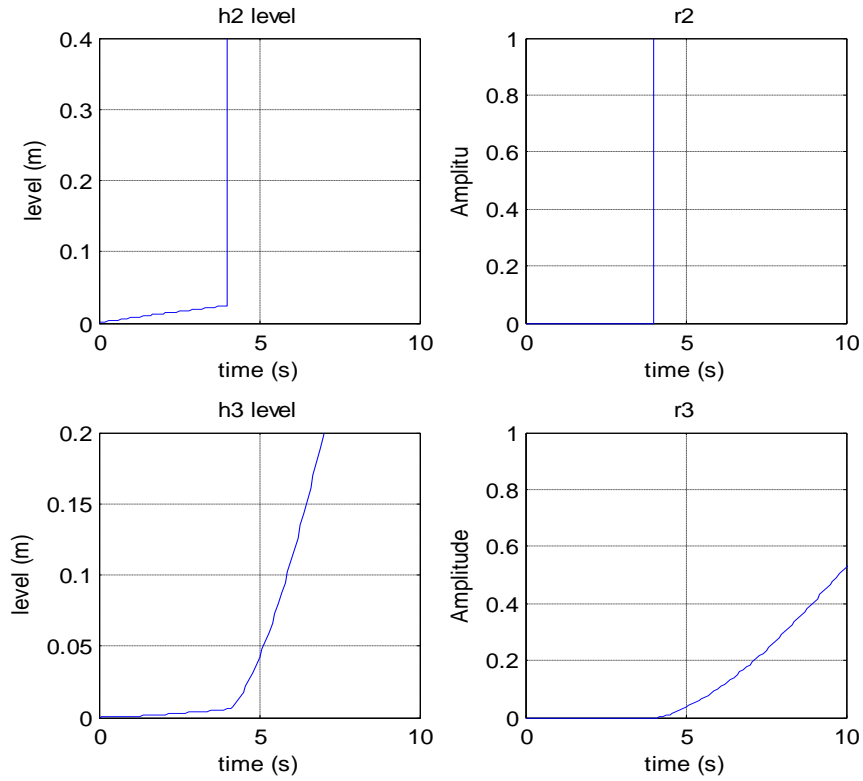


Figure 8: Output system and residuals in the presence of a fault in actuator U2

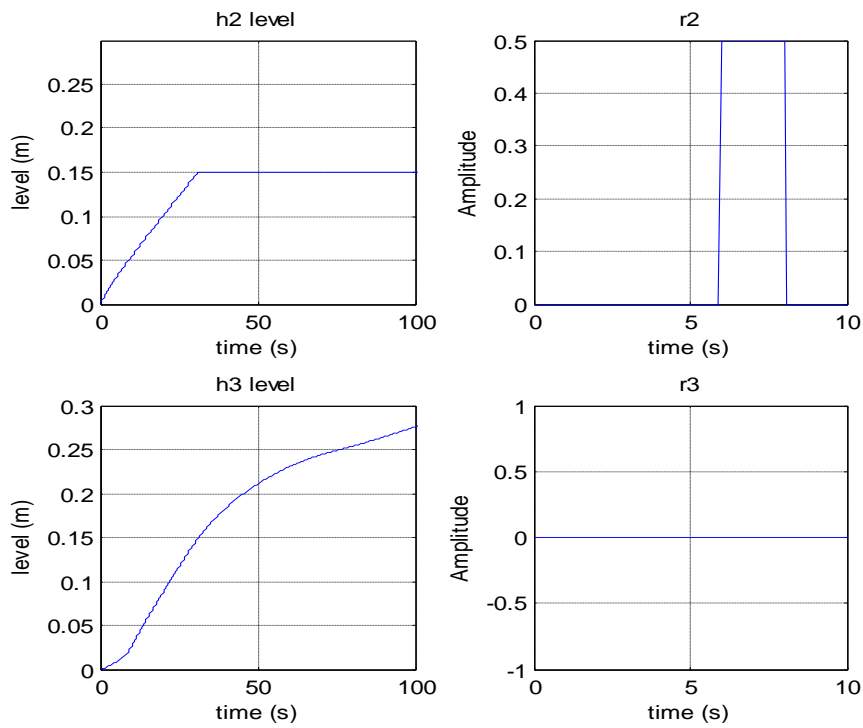


Figure 9: Output system and residuals in the presence of a fault in the sensor  $h_2$

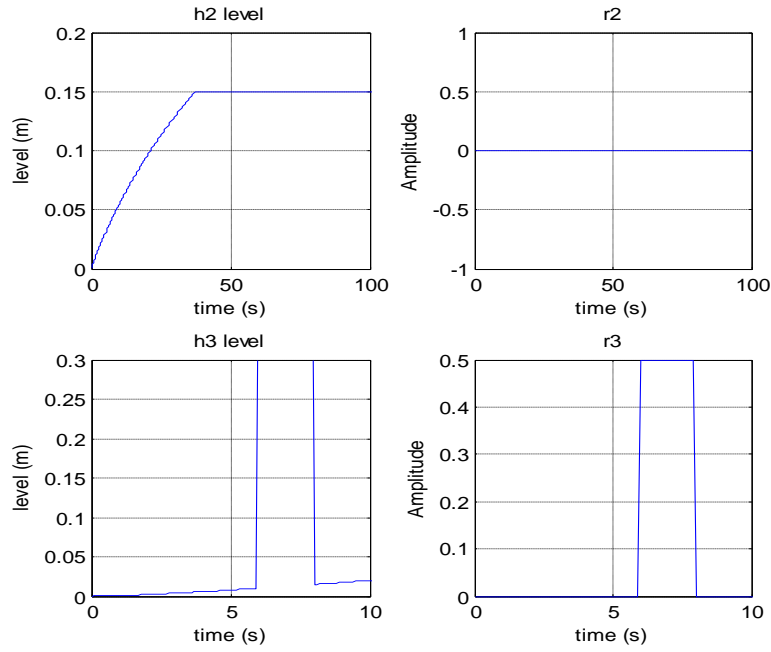


Figure 10: Output system and residue in the presence of a fault in the sensor  $h_3$

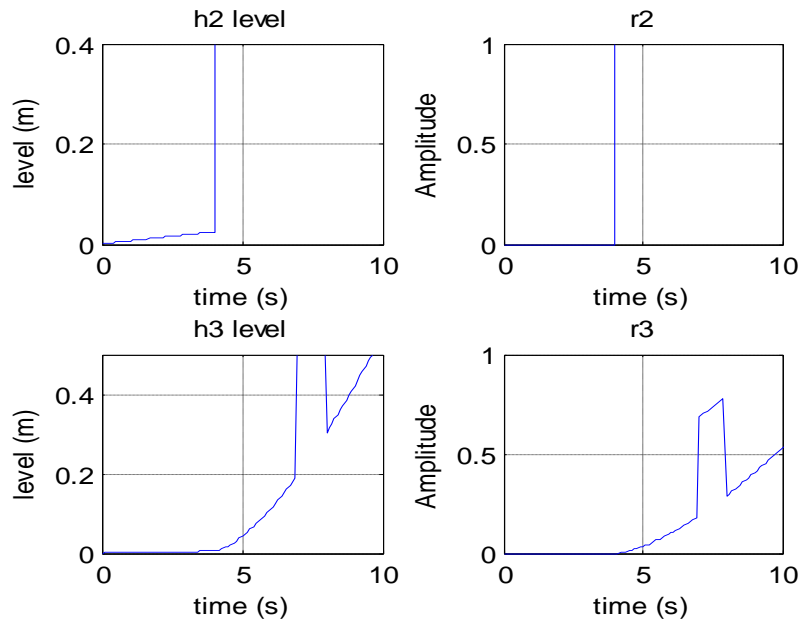


Figure 11: Output system and residuals in the presence of a fault in the sensor  $h_3$  and in the actuator  $U_2$  simultaneously

## VI CONCLUSION

This paper presents the simple approach to detect single and simultaneous faults of dynamic systems. This method is applied to a process of three tanks.

The model of the water tank has been implemented in the Matlab/Simulink environment. The simulation results confirm the robustness and effectiveness of the proposed approach for fault detection in the presence of multiple defaults.

## VII ACKNOWLEDGEMENTS

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