Research Inventy: International Journal of Engineering and Science
Vol.16, Issue 4 (April 2026), PP 52-58
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Robust H_∞ control for automatic stabilization systems on missiles with uncertain parameters.

[1] Nguyen Thanh Chung, [2] Dang Cong Vu
Faculty of Aeronautical Engineering, Academy of Air Force-Air Defense.

Abstract: *During motion, a missile is a controlled object whose parameters vary over a wide range, such as aerodynamic coefficients, mass, velocity, and moments of inertia. The unknown variation of these parameters causes significant difficulties in controller synthesis. This paper presents the results of synthesizing an robust* H_∞ *controller for the pitch autopilot in the vertical plane of a conventional aerodynamic configuration missile, taking into account variations in mass, moment of inertia, and aerodynamic coefficients. The performance and robustness of the missile pitch autopilot are verified through nonlinear simulation results using a NASA sample missile model.*
Keywords: *Automatic stabilization system on the missile; missile autopilot system; Robust* H_∞ *control.*

Date Of Submission: 08-04-2026 Date of Acceptance: 21-04-2026

I. INTRODUCTION

A control system always contains uncertainties. The sources of these uncertainties may be: (i) the plant is described incompletely, or the model describing the plant is inaccurate; or (ii) the plant model we use is a simplified model, approximated from a complex model that accurately describes the plant. For example, the actual plant may have an exact nonlinear model, but to simplify the analysis and controller design, we approximate that nonlinear model by a linear model.

To address the problem of ensuring robust controllability for plants with uncertain parameters, numerous research works have applied various control theories. Among them, the robust pole placement technique has been used to design missile autopilot systems [1]. The method based on Linear Matrix Inequalities (LMIs) combined with gain-scheduling has been applied to design an autopilot system in order to handle the variation of mass during missile flight [2]. The Linear Parameter Varying (LPV) approach has been employed to design an adaptive controller for the altitude channel autopilot [3]. Alternatively, the dynamic inversion method can be applied to design a nonlinear controller to ensure missile robustness [4]. In [5], the authors designed a missile autopilot with uncertain parameters using the robust backstepping method; the results ensured robustness while maintaining tracking performance. Since the 1980s, the H∞ method and synthesis techniques have been used to design controllers for systems with multiple uncertainties [6]. These controllers have been widely applied in missile control, as they guarantee robustness and performance requirements in the presence of uncertainties and disturbances. In [7], a dynamic inversion controller was used for the inner loop, while an H∞ controller was employed for the outer loop to achieve robust performance.

Using the application-oriented approach of H∞ robust control, this paper presents the results of applying the loop-shaping technique to synthesize a robust controller for the automatic stabilization system in a missile compartment. The structure of the paper is organized as follows: After the introduction, the nonlinear dynamic model and the linearized model of the missile in the vertical plane are presented. Next, the H∞ loop-shaping robust control method is briefly introduced as the basis for synthesizing a robust controller for the missile's automatic stabilization system, taking into account parameter variations. Finally, the paper presents the controller synthesis results and evaluates the controller performance through nonlinear simulations in the Matlab environment.

II. THE MISSILE DYNAMIC MODEL IN THE VERTICAL PLANE

Consider a missile with a conventional aerodynamic configuration consisting of two pairs of wings symmetric about the longitudinal axis. It is assumed that the missile is stably controlled about the longitudinal axis and that there is no cross-coupling between the control planes. The rotational motion of the missile in the vertical plane of the body-fixed coordinate system OX_1Z_1 can be described by the following system of equations (1) [8]:

$$\dot{\alpha}=\frac{q_\infty S_{\text{ref}}}{mV_M}\left(\sin\alpha C_A-\cos\alpha C_N\right)+q-\sin\alpha\frac{q_\infty S_{\text{ref}}C_T}{mV_M}$$
$$\dot{q}=\frac{q_\infty S_{\text{ref}}d}{I_y^b}\left(C_m+\bar{s}\cdot C_N\right)$$

where α is the angle of attack of the missile; q is the pitch angular rate of the missile; q_∞ is the dynamic pressure; S_{ref} is the reference area of the missile; m is the missile mass; V_M is the missile velocity; d is the missile diameter; $\bar{s}$ is the normalized distance (with respect to the missile diameter) from the missile center of mass to the center of pressure. $I_{y_1}^b$ is the moment of inertia of the missile about the Oy_1 axis of the body-fixed coordinate system. C_A, C_N, C_T, and C_m denote the axial force coefficient, normal force coefficient, thrust coefficient, and pitching moment coefficient, respectively. The coefficients C_N, C_m, and C_A are approximated by Eqs.~(2), (3), and (4), respectively.

$$C_N=C_{N\alpha}\alpha+C_{N\alpha|\alpha|}\alpha|\alpha|+C_{N\alpha^3}\alpha^3+\left(C_{N\delta_q}+C_{N\alpha\delta_q}\alpha\right)\delta_q+\left(C_{Nq}q+C_{N\dot{\alpha}}\dot{\alpha}\right)\frac{d}{2V_M} \quad (1)$$

$$C_m=C_{m\alpha}\alpha+C_{m\alpha|\alpha|}\alpha|\alpha|+C_{m\alpha^3}\alpha^3+C_{m\delta_q}(\alpha)\delta_q+\left(C_{mq}q+C_{m\dot{\alpha}}\dot{\alpha}\right)\frac{d}{2V_M} \quad (2)$$

$$C_A=C_{A_0}+C_{A|\alpha|}|\alpha|+C_{A\alpha^2}\alpha^2+C_{A|\alpha|^3}|\alpha|^3+\Delta C_{Ab}+\left(C_{A\delta_q}\operatorname{sgn}\delta_q+C_{A\alpha\delta_q}\alpha+C_{A\alpha^2\delta_q}\alpha^2\right)\delta_q \quad (3)$$

Substituting the aerodynamic coefficients (2), (3), and (4) into equation (1), we obtain:

$$\begin{cases}\dot{\alpha}=a_{11}^a\alpha+a_{12}^a q+b_{11}^a\delta_q\\ \dot{q}=a_{21}^a\alpha+a_{22}^a q+b_{22}^a\delta_q\end{cases} \quad (4)$$

Where:

$$a_{11}^a=\frac{q_\infty S_{ref}}{mV_M D_{\dot{\alpha}}}\Big[\sin\alpha\left(C_{A\alpha}+C_{A\alpha^2}|\alpha|+C_{A\alpha^3}\alpha^2\right)\operatorname{sgn}\alpha+\operatorname{sinc}\alpha\left(C_{A_0}+\Delta C_{Ab}-C_T\right)$$
$$-\cos\alpha\left(C_{N\alpha}+C_{N\alpha|\alpha|}|\alpha|+C_{N\alpha^3}\alpha^2\right)\Big];$$

$$\operatorname{sinc}\alpha=\begin{cases}\dfrac{\sin\alpha}{\alpha}, & \alpha\neq 0\\ 1, & \alpha=0\end{cases};\qquad a_{12}^a=\frac{1}{D_{\dot{\alpha}}}\left[1-\frac{q_\infty S_{ref}d}{2mV_M^2}\cos\alpha C_{N_q}\right];$$

$$a_{21}^a=\frac{q_\infty S_{\text{ref}}d}{I_y^b}\left[C_{m\alpha}+C_{md|\alpha|}|\alpha|+C_{m\alpha^3}\alpha^2+\bar{s}\left(C_{N\alpha}+C_{N\alpha|\alpha|}|\alpha|+C_{N\alpha^3}\alpha^2\right)\right];$$

$$a_{22}^a=\frac{q_\infty S_{ref}d^2}{2V_M I_y^b}\left(C_{mq}+\bar{s}C_{Nq}\right);$$

$$b_{11}^a=\frac{q_\infty S_{ref}}{mV_M D_{\dot{\alpha}}}\left[\left(\sin\alpha C_{A\delta_q}\operatorname{sgn}\delta_q-\cos\alpha C_{N\delta_q}\right)+\alpha\left(\sin\alpha C_{A\alpha\delta_q}-\cos\alpha C_{N\alpha\delta_q}\right)+\sin\alpha C_{A\alpha^2\delta_q}\alpha^2\right]$$

$$b_{21}^a=\frac{q_\infty S_{ref}d}{I_y^b}\left[C_{m\delta_q}(\alpha)+\bar{s}\left(C_{N\delta_q}+\alpha C_{N\alpha\delta_q}\right)\right];\qquad D_{\dot{\alpha}}=1+\frac{q_\infty S_{ref}}{mV_M}\cos\alpha\frac{d}{2V_M}C_{N\dot{\alpha}};$$

δ_q is the control surface deflection angle; sgn(·) is the sign function.

Writing equation (5) in matrix form, we obtain:

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}a_{11}^a & a_{12}^a\\ a_{21}^a & a_{22}^a\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix}+\begin{bmatrix}b_{11}^a\\ b_{21}^a\end{bmatrix}\delta_q \quad (6)$$

Let:

$$\mathbf{x}=\begin{bmatrix}\alpha & q\end{bmatrix}^T;\ \mathbf{u}=\delta_q;\ \mathbf{y}=\begin{bmatrix}\alpha & q\end{bmatrix}^T$$

$$\mathbf{A}_a=\begin{bmatrix}a_{11}^a & a_{12}^a\\ a_{21}^a & a_{22}^a\end{bmatrix};\ \mathbf{B}_a=\begin{bmatrix}b_{11}^a\\ b_{21}^a\end{bmatrix};\ \mathbf{C}_a=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix};\ \mathbf{D}_a=\mathbf{0}$$

Thus, the missile's dynamic equations in the vertical plane are represented in state-space form as follows (7):

$$\begin{cases}\dot{\mathbf{x}}=\mathbf{A}_a\mathbf{x}+\mathbf{B}_a\mathbf{u}\\ \mathbf{y}=\mathbf{C}_a\mathbf{x}+\mathbf{D}_a\mathbf{u}\end{cases} \quad (7)$$

The missile dynamic model in the vertical plane is linearized at specific flight conditions and operating states of the missile based on the calculation of the coefficients in equation (7).

The missile dynamic model in the vertical plane is linearized at specific flight conditions and operating states of the missile based on the calculation of the coefficients of the system of equations (7).

III. ROBUST CONTROLLER SYNTHESIS AND PERFORMANCE EVALUATION

A control system is considered robust if it remains stable and satisfies certain performance criteria in the presence of uncertainties. The objective of robust control synthesis is to design a controller for a given system such that the resulting closed-loop system is robust.

The optimization method has been proven to be an effective and robust design approach for linear time-invariant control systems. Although this method is effective, the representation of model uncertainties is limited by constraints on the number of poles in the right-half complex plane. In addition, undesirable pole-zero cancellations may occur between the nominal model and the controller [9].

Therefore, this paper employs the loop-shaping design method, which helps overcome limitations related to the number of right-half-plane poles and avoids unwanted pole-zero cancellations between the nominal model and the designed controller. This method also does not require an iterative procedure to obtain an optimal controller, thereby improving computational efficiency.

Neglecting the dynamics of the sensors and assuming that the angle of attack and pitch rate are measurable, the structure of the missile autopilot stabilization system is represented in **Figure 1**.

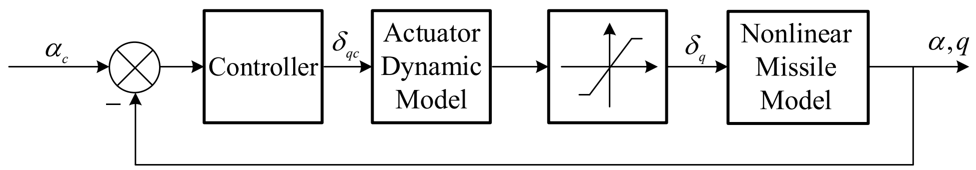

Figure 1. Structure diagram of the missile onboard automatic stabilization system

The dynamic model of the actuator system is approximated as a second-order oscillatory element as follows:

$$\begin{bmatrix} \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_a^2 & -2\zeta_a\omega_a \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_a^2 \end{bmatrix} \delta_{qc} \tag{8}$$

Where δ_{qc} and δ are respectively the input and output signals of the actuator model.

$$\zeta_a = 0.707, \quad \omega_a = 150 \text{ rad/s.}$$

Considering the missile model, a conventional aerodynamic configuration with geometric parameters shown in Figure~2 and aerodynamic coefficients given in [8] is used.

The angle of attack is limited by $|\alpha| \le 30^\circ$, and the control surface deflection angle satisfies $|\delta_q| \le 30^\circ$.

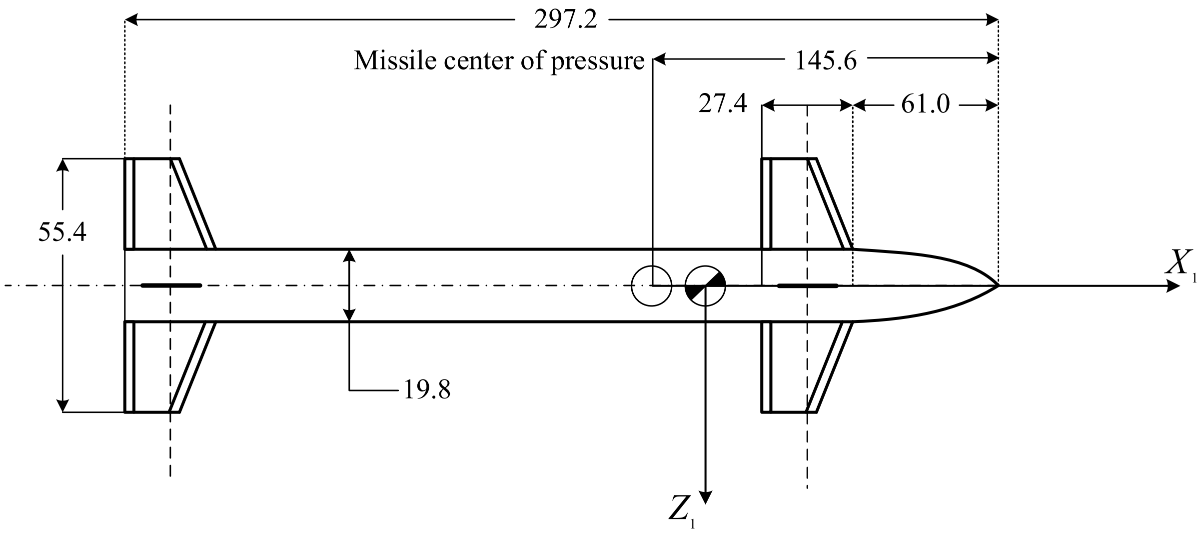

Figure 2. Representative missile model

Some main parameters of the missile are presented in **Table 1**.

Table 1. Main parameters of the missile

Parameter	Unit	Value	Parameter	Unit	Value
Altitude	m	6000	Velocity	m/s	790 (2.5M)
Center of gravity	m	1.288 – 1.436	Mass	kg	87.27 – 101.3
Reference area	dm²	3.0828	Moment of inertia	kg·m²	32 – 33.2

From the system of equations (7), the linearized missile model at the boundary values of parameters such as mass, moment of inertia, center of gravity position, angle of attack, and control surface deflection is computed and represented in the form of an uncertain parameter model as follows:

$$\begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} (-4,2706;\ -2,4625) & 0,9995 \\ (-2660,5;-527,47) & -9,3868 \end{bmatrix}\mathbf{x} + \begin{bmatrix} (-0,6319;\ 0,0224) \\ (-4242,3;-2135,4) \end{bmatrix}\mathbf{u} \\ \mathbf{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{x} \end{cases} \tag{9}$$

The transient response characteristics of the linearized missile model under parameter variations are illustrated in Figure 3.

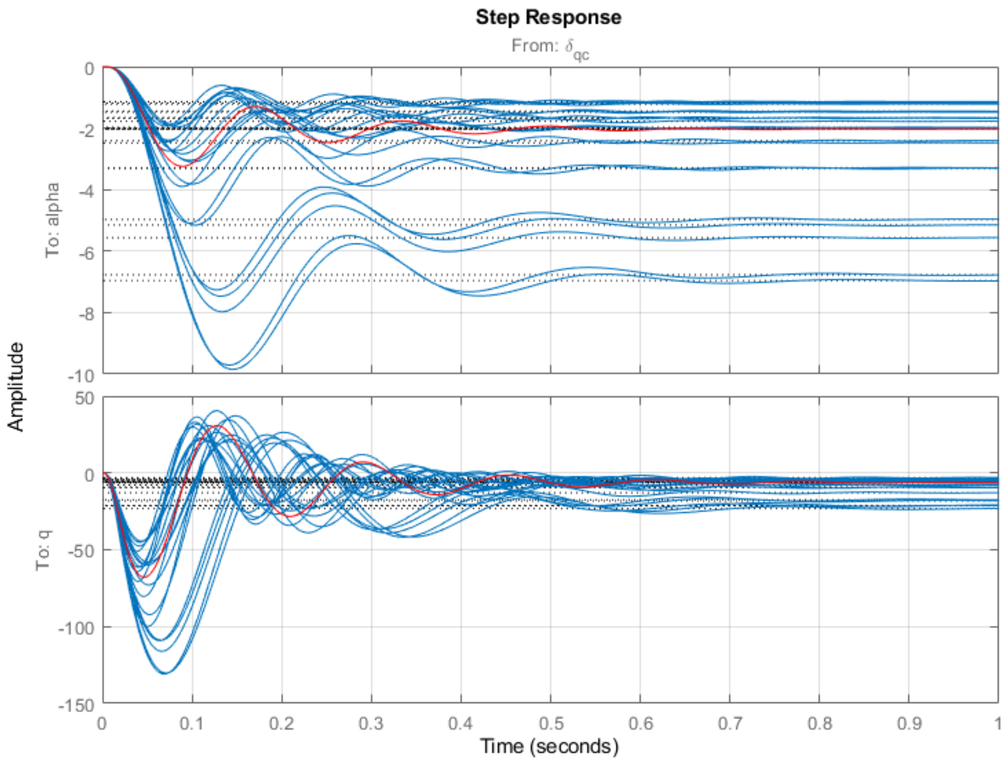

Figure 3. Transient response of the linearized missile model under parameter variations

Using the H_∞loop-shaping design method for the uncertain plant (9), the system response shown in Figure 4 is obtained. The controller stabilizes the system against parameter variations of the linearized missile model within the considered range.

The simulation results for the nonlinear missile model (Figure 5) show that the missile's angle of attack tracks the commanded angle of attack with good performance, ensuring small steady-state error and short transient time. When the commanded angle of attack is large, the steady-state error and transient time increase due to the reduced effectiveness of the aerodynamic control surfaces.

Figure 6 presents the control surface deflection angle of the missile. When the commanded angle of attack is large (+30 degrees), a significant steady-state error (about 2 degrees) appears because the control surface deflection reaches its saturation limit.

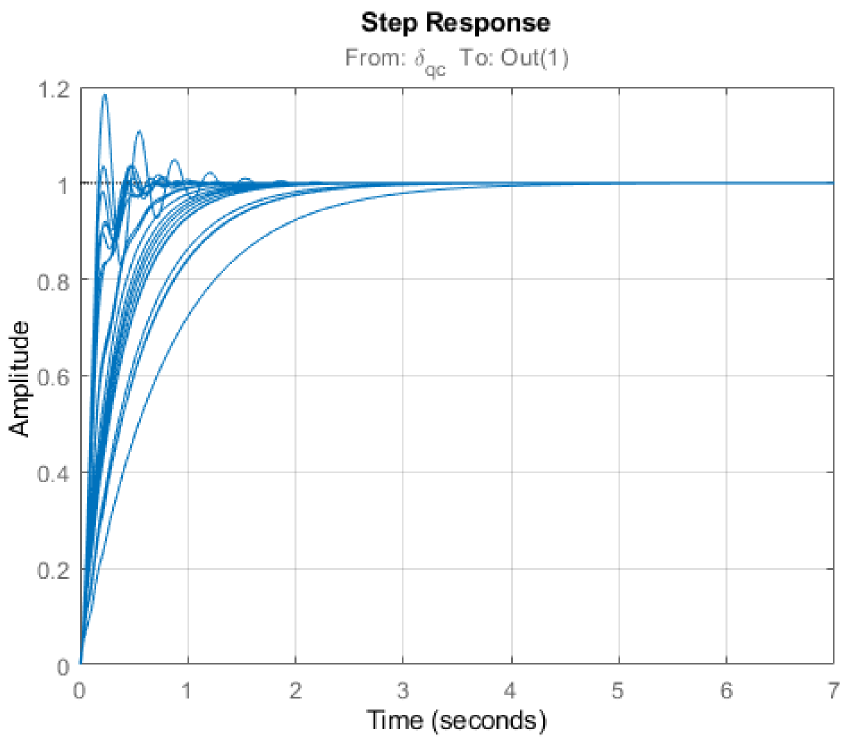

Figure 4. Transient response of the nonlinear missile model with H_∞control

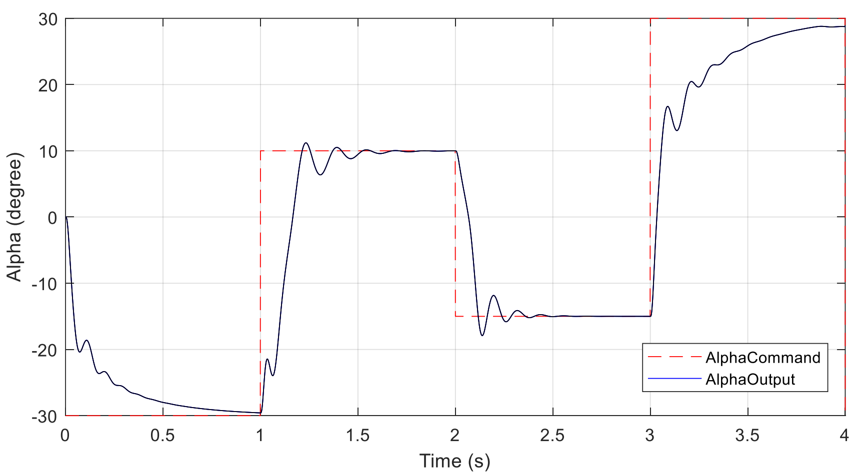

Figure 5. Angle of attack of the missile in nonlinear model simulation

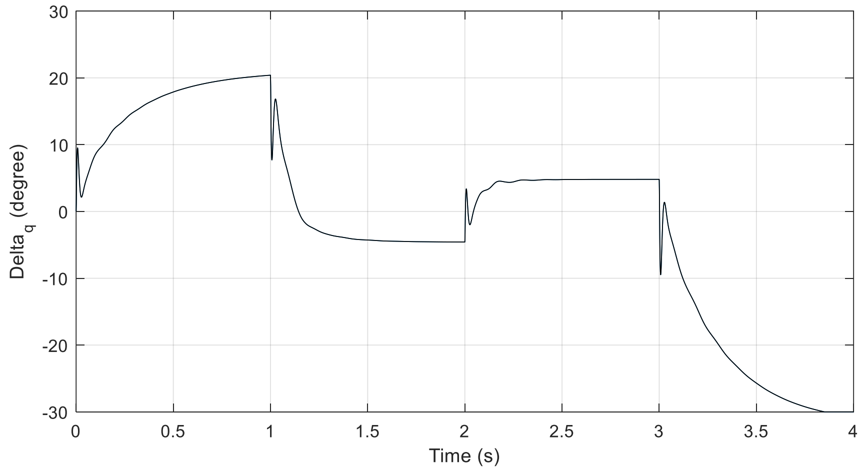

Figure 6. Control surface deflection angle of the missile in nonlinear model simulation

IV. CONCLUSION

This paper presents the results of designing an robust controller for the missile onboard automatic stabilization system with uncertain parameters. The controller is synthesized based on the linearized missile model in the vertical plane with varying parameters and then evaluated using the nonlinear model. The nonlinear simulation results confirm that the controller achieves good tracking performance and robustness against parameter variations. Based on these results, the control performance can be further improved by combining the proposed approach with gain-scheduling or fuzzy logic methods to account for variations in missile velocity and flight altitude.

REFERENCES

[1]. Ryu, S.-M., Won, D.-Y., Lee, C.-H., Tahk, M.-J. J. I. J. o. A., & Sciences, S. (2011). Missile autopilot design for agile turn control during boost-phase. *12*(4), 365-370.

[2]. Won, D.-Y., Tahk, M.-J., & Kim, K.-Y. (2010). *Robust gain-scheduling technique for an agile missile subject to mass variation.* Paper presented at the ICCAS 2010.

[3]. Theodoulis, S., Duc, G. J. J. o. g., control,, & dynamics. (2009). Missile autopilot design: gain-scheduling and the gap metric. *32*(3), 986-996.

[4]. Liu, Z., Cao, B., Liang, X., & Jia, X. (2006). *A composite dynamic inverse controller design for agile missile.* Paper presented at the 2006 6th World Congress on Intelligent Control and Automation.

[5]. Lee, S., Kim, Y., Lee, Y., Moon, G., Jeon, B.-E. J. I. T. o. A., & Systems, E. (2020). Robust-backstepping missile autopilot design considering time-varying parameters and uncertainty. *56*(6), 4269-4287.

[6]. Doyle, J., Glover, K., Khargonekar, P., & Francis, B. (1988). *State-space solutions to standard H 2 and H∞ control problems.* Paper presented at the 1988 American Control Conference.

[7]. Sadraey, M., & Colgren, R. (2006). *Robust nonlinear controller design for a complete UAV mission.* Paper presented at the AIAA Guidance, Navigation, and Control Conference and Exhibit.

[8]. Hernández, F. d. B. I. (2015). *Optimization of the Integrated Guidance and Control for a Dual Aerodynamic Control Missile.* Universidad Politécnica de Madrid,

[9]. Sefton, J., Glover, K. J. S., & Letters, C. (1990). Pole/zero cancellations in the general H∞ problem with reference to a two block design. *14*(4), 295-306.