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Determination of the Transition Point at the End of the Autonomous Phase to Guided Mode Satisfying Predefined Conditions

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Abstract: *This paper presents an algorithm for determining the desired transition point from the autonomous flight phase to the homing phase while satisfying predefined conditions. The requirements at the end of the autonomous flight phase include ensuring the appropriate homing distance and aligning the missile velocity vector with the missile-target line of sight. The analysis is conducted for a target moving with constant velocity and different initial heading angles, where the missile trajectory follows a Rayleigh function. The results indicate that the proposed algorithm meets the specified requirements.*

Keywords: *Autonomous Flight Phase, Transition Point, Missiles guidance, Rayleigh function.*

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I. INTRODUCTION

In air-to-air combat, to ensure the capability to neutralize targets while maintaining safety for the carrier aircraft and ease of use for pilots employing the "fire-and-forget" method, there has been a strong research focus by the U.S., Russia, and Europe on improving the accuracy of air-to-air missiles capable of engaging beyond-visual-range (BVR) targets (outside the seeker head's observation area). The guidance method for these missiles typically employs a combination approach.

The first phase is autonomous flight to ensure long-range coverage and large initial interception angles. The second phase uses self-guidance to achieve high accuracy. Among these, the first phase is critical for the overall success of the second phase in neutralizing the target.

The problem of synthesizing optimal guidance laws during the autonomous flight phase has been addressed in [5] and related published works. However, to close the problem, it is necessary to determine the desired endpoint of the autonomous flight phase, ensuring favorable conditions for transitioning to the self-guidance phase.

For the class of medium-range air-to-air missiles employing the "autonomous-self-guidance" method currently deployed in the Vietnam People's Air Force, the autonomous flight phase involves launching the missile towards a desired endpoint determined via the target designation system. However, the algorithm for determining this desired endpoint has not been clearly defined in the technical documentation.

This study aims to develop an algorithm to solve the problem of determining the desired endpoint (the interception problem) that ensures the missile can follow an arbitrary curved trajectory during the autonomous flight phase. Simultaneously, it satisfies the requirements for stealthy target approach, thereby enhancing the combat effectiveness of medium-range air-to-air missiles.

II. TRAJECTORY OPTIMIZATION AND ALGORITHM FOR SOLVING THE INTERCEPTION PROBLEM

2.1 Basis for Building the Trajectory Optimization Problem

The missile's motion trajectory is constructed in the horizontal plane of the launch coordinate system XOZ as an arbitrary curve, as illustrated in Figure 1.
In this context:
- X_m, Z_m, V_m, ψ_{ms} correspond to the missile's coordinates, velocity, and trajectory heading angle;
- X_{t0}, Z_{t0}, V_t, ψ_t correspond to the initial coordinates, velocity, and trajectory heading angle of the target;
- X^*_m, Z^*_m, ψ^*_m correspond to the missile's coordinates and trajectory heading angle at the moment of transitioning to self-guidance;
- ψ_b is the compensation angle.

These parameters are determined in Section 2.3;

Point M_0 has coordinates $(x_0,0)$ where

$$x_0 = x^*_m - z^*_m(\cos\psi^*_m / \sin\psi^*_m)$$

$M_0x'z'$: A local Cartesian coordinate system attached to M_0 with a trajectory heading angle ψ^*_m

The problem of missile guidance during the autonomous flight phase is to determine the required overload such that the missile trajectory originating from point O passes through point M, with the velocity vector at M aligned with ψ^*_m. The missile trajectory follows an arbitrary curve.

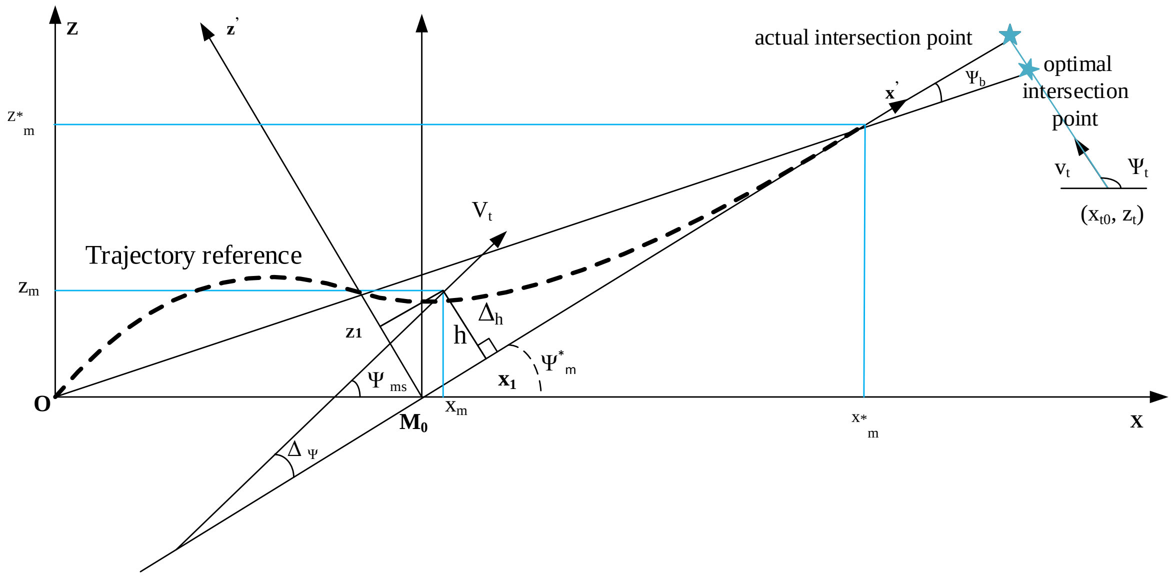

Figure 1. Position of the missile relative to the reference trajectory

2.2. Buliding of the Missile Kinematics Model

Consider the motion of the missile in the $M_0x'z'$ plane. The kinematic equations of the missile are given by [1, 2, 3]:

$$\begin{cases} \Delta\dot{\psi} = (g/V_m)n_z \\ \dot{x}_1 = V_m\cos\Delta\psi \\ \dot{z}_1 = V_m\sin\Delta\psi \end{cases} \quad (1)$$

The symbols (x_m, z_m) and (x_1, z_1) represent the coordinates of the reference trajectory and the missile at time t, respectively, in the $M_0x'z'$ coordinate syste. Additionally, the reference trajectory can be expressed in the $M_0x'z'$ coordinate system in the form: $z_m=f(x')$.

The function $f(x')$ must satisfy the following conditions: it passes through the origin O, asymptotically approaches the M_0x' axis, and its trajectory can vary flexibly depending on the coefficients of the function (the rule of the function will be selected in section 2.2). The deviation between the missile trajectory and the reference trajectory is determined as follows: $\Delta h = h - z_m = z_1 - z_m$

Taking the derivative on both sides, we obtain:

$\Delta\dot{h} = \dot{z}_1 - \dot{z}_m = \dot{z}_1 - \frac{\partial f(x_1)}{\partial x_1}\dot{x}_1$ The symbol $f_x = \partial f(x_1)/\partial x_1$ substituting $\dot{x}_1$, $\dot{z}_1$ from system equation (1), we obtain:

$$\Delta\dot{h} = V_m\sin\Delta\psi - f_xV_m\cos\Delta\psi$$

$$\Delta\dot{h} = V_m\sqrt{1+f_x^2}\left(\frac{1}{\sqrt{1+f_x^2}}\sin\Delta\psi - \frac{f_x}{\sqrt{1+f_x^2}}\cos\Delta\psi\right) \Leftrightarrow \Delta\dot{h} = V_m\sqrt{1+f_x^2}\sin(\Delta\psi - \psi_m)$$

In there:

$$\sin\psi_m = \frac{f_x}{\sqrt{1+f_x^2}};\ \cos\psi_m = \frac{1}{\sqrt{1+f_x^2}};\ \tan\psi_m = f_x$$

We have ψ_m as the inclination angle of the reference trajectory at time *t*.
Set $\Delta\bar{\psi} = \Delta\psi - \psi_m$, with $\Delta\psi$ as the angular deviation of the missile trajectory from the inclination angle of thereference trajectory. Taking the derivative on both sides, we obtain:

$$\Delta\dot{\bar{\psi}} = \Delta\dot{\psi} - \dot{\psi}_m \tag{3}$$

Taking the derivative on both sides $\tan\psi_m = f_x$ we obtain:

$$\dot{\psi}_m = f_{xx}\cos^2\psi_m V_t \cos\Delta\psi \quad \text{with} \quad f_{xx} = \frac{\partial^2 f(x_1)}{\partial x_1^2} \tag{4}$$

$$\frac{1}{\cos^2\psi_m}\dot{\psi}_m = f_{xx}\dot{x}_1 = f_{xx}V_m\cos\Delta\psi$$

Substituting (4) into (2) and (3), and combining with the first equation of system (1), we obtain

$$\begin{cases} \Delta\dot{\bar{\psi}} = \dfrac{g}{V_m} n_z - \dot{\psi}_m \\ \Delta\dot{h} = V_m\sqrt{1+f_x^2}\sin(\Delta\bar{\psi}) \end{cases} \tag{5}$$

Set: $$n_{zm} = \sqrt{1+f_x^2}\,(n_z - \frac{V_m}{g}\dot{\psi}_m);\ \bar{V}_m = V_m\sqrt{1+f_x^2}$$

System (5) become:

$$\begin{cases} \Delta\dot{\bar{\psi}} = (g/\bar{V}_m)n_{zm} \\ \Delta\dot{h} = \bar{V}_m\sin(\Delta\bar{\psi}) \end{cases} \tag{6}$$

Boundary condition at $t_f : \Delta\bar{\psi}_{t_f} \to 0;\ \Delta h_{t_f} \to 0$ *with* $t_f < t_{td}$.

****) Constructing guidance law***

Selecting a quality index function of the form [4]:

$$J = \frac{1}{2}\rho_1\Delta\bar{\psi}_{tf}^2 + \frac{1}{2}\rho_2\Delta h_{tf}^2 + \frac{1}{2}\int_{t_0}^{t_f} n_{zm}^2 dt \to \min \tag{7}$$

From (7), obtain the Terminant function:

$$G = \frac{1}{2}\begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f}^T \left[P^f\right]_{2x2}\begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f},\ P^f = \begin{bmatrix}\rho_1 & 0 \\ 0 & \rho_2\end{bmatrix} \tag{8}$$

Haminton function:

$$H = \lambda_{\Delta\bar{\psi}}\frac{g}{V_m}n_{zm} + \lambda_{\Delta h}\overline{V_m}\sin\Delta\bar{\psi} + \frac{1}{2}n_{zm}^2 \tag{9}$$

The optimal solution is determined by the expression $\dfrac{\partial H}{\partial n_z} = 0$

$$n_{zm} = -\frac{g}{\bar{V}_m}\lambda_{\Delta\bar{\psi}} \tag{10}$$

From the selected performance index function, according to [5], the optimal guidance law is derived based on angular deviation and lateral deviation for a trajectory of any arbitrary curve:

$$n_z = -\frac{k_1\Delta\bar{\Psi} + k_2\Delta h}{\sqrt{1+f_x^2}} + \frac{V_m}{g}\dot{\psi} \tag{11}$$

With $$k_1 = \frac{n_{zm\text{-}CP} + \frac{1}{gT_{CP}^2}\Delta h_{\max}}{\frac{1}{V_m T_{CP}}\Delta h_{\max} + \Delta\psi_{\max}}; \qquad k_2 = \frac{n_{zm\text{-}CP} + \frac{\overline{V}_m}{gT_{CP}}\Delta\overline{\psi}_{\max}}{\Delta h_{\max} + \overline{V}_m T_{CP}\Delta\overline{\psi}_{\max}}$$

In the guidance law (11), it is observed that:
The first term is used to eliminate the deviation of the missile trajectory from the reference trajectory based on angular deviation and linear deviation.
The second term represents the required acceleration, which is determined according to the reference trajectory.

$$f_x^2 = 0; \quad \dot{\psi}_m = f_{xx}\cos^2\psi_m V_m \cos\Delta\psi = 0$$

***) Selection of Reference Function**

From a theoretical perspective, a straight-line trajectory is the optimal path for the missile from launch to the end of the autonomous flight phase. However, considering operational conditions and real-world tactical requirements, the missile must be capable of:
Executing attacks with diverse trajectory profiles;
Engaging targets with a large initial intercept angle q_0.
To accommodate variations in initial launch positions and ensure that the trajectory amplitude aligns with practical combat scenarios, a modified Rayleigh function is formulated as follows

$$z = K\frac{x-\overline{x}}{\delta^2}e^{-\frac{(x-\overline{x})^2}{2\delta^2}} \tag{12}$$

Function (12) has the OX axis as a horizontal

$$z_{\max} = (K/\delta)e^{-\frac{1}{2}}$$

asymptote. At $x = \overline{x}$ function z=0; at $x - \overline{x} = \delta$ the function reaches its maximum value, with the corresponding maximum value denoted accordingly .
Function (12) is represented in both the ground coordinate system and the coordinate system aligned with the desired heading angle, as illustrated in Figure 2.

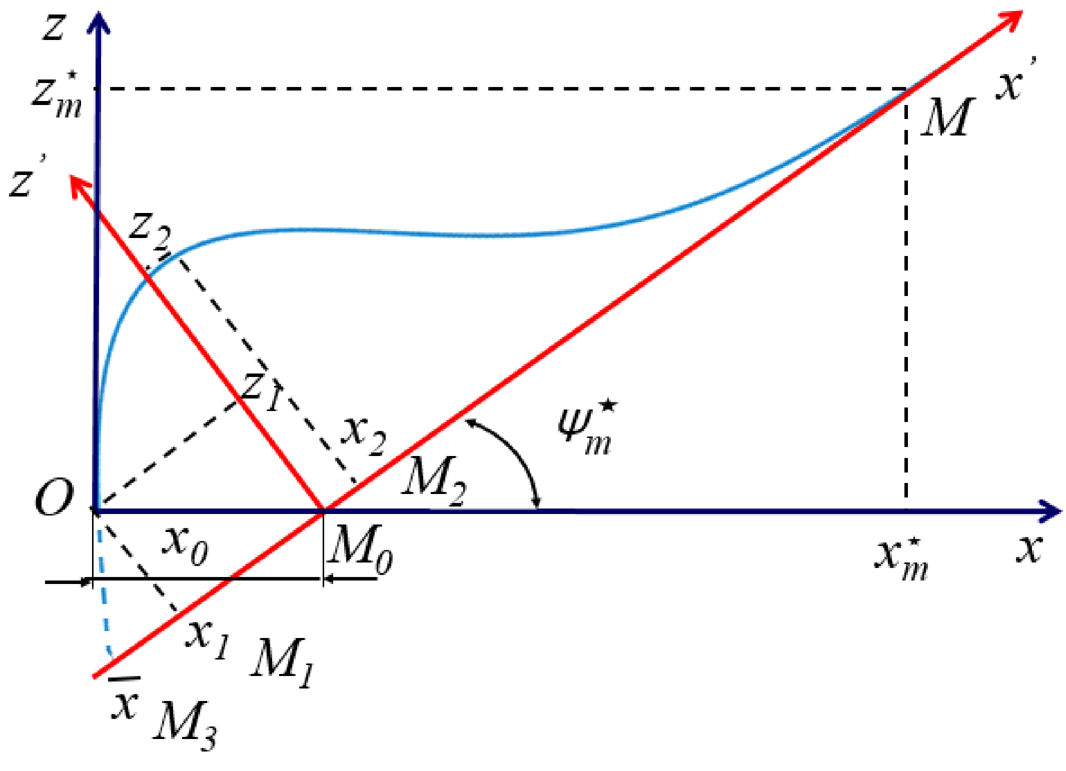

Figure 2. Graph of the Rayleigh Function in the Moving Ground Coordinate System

To apply function (12) in constructing the reference trajectory, the requirement is to determine K and δ a parameter such that the function passes through $M_1(x_1, z_1)$ and reaches its maximum at $M_2(x_2, z_2)$. These points (x_1, z_1) , (x_2,z_2) are predefined in the given coordinate system $M_0x'z'$(ψ_m are predefined).

There are $z_{max} = (K/\delta)e^{-\frac{1}{2}} = z_2$ obtained

$$\frac{K}{\delta} = z_2 e^{\frac{1}{2}} \tag{13}$$

The function passes through the point (x_1,z_1), and has substituting $x - \bar{x} = \delta$ into (12) we have:

$$z_1 = K\frac{x_1 - \bar{x}}{\delta^2}e^{\frac{(x_1-x)^2}{2\delta^2}} = K\frac{x_1 - x_2 + \delta}{\delta^2}e^{\frac{(x_1-x_2+\delta)^2}{2\delta^2}} \tag{14}$$

Substituting (13) into (14) obtained:

$$\frac{x_1 - x_2 + \delta}{\sqrt{2}\delta}e^{\frac{-(x_1-x_2+\delta)^2}{2\delta^2}} = \frac{z_1}{\sqrt{2}z_2}e^{-\frac{1}{2}} \tag{15}$$

Using the bisection method, solve equation (15) with the variable $\frac{x_1 - x_2 + \delta}{\sqrt{2}\delta}$ finding the root,

$\frac{x_1 - x_2 + \delta}{\sqrt{2}\delta} = x^*$ for $0 < x^* < 1$;

Finally obtained K, δ need find:

$$\delta = \frac{x_1 - x_2}{1 - \sqrt{2}x^*}, \qquad K = z_2 e^{1/2}\delta \tag{16}$$

At the same time, get the expression to calculate the first partial derivative with respect to x:

$$f_x = \frac{K}{\delta^2}e^{-\frac{(x-\bar{x})^2}{2\delta^2}}\left(1 - \left(\frac{x-\bar{x}}{\delta}\right)^2\right) \tag{17}$$

Similarly, the second derivative gets:

$$f_{xx} = \frac{K}{\delta^2}\frac{x-\bar{x}}{\delta^2}\left(3 - \frac{\overline{(x-x)}^2}{\delta^2}\right)e^{-\frac{(x-\bar{x})^2}{2\delta^2}} \tag{18}$$

In this context, Equation (17) is used to determine the normal acceleration z_n while expression (18) is used to determine the rate of change of the heading angle $\dot{\psi}_m$ in the guidance law (11).

2.3. Developing an Algorithm for Interception Guidance

The guidance law (23), with a reference trajectory following a Rayleigh function, requires determining the desired target coordinates $\left(x_m^*, z_m^*\right)$, and the desired heading angle ψ_m^*. These parameters $\left(x_m^*, z_m^*\right)$, ψ_m^* at the initial moment of autonomous guidance are determined by solving the interception problem based on target indication information.

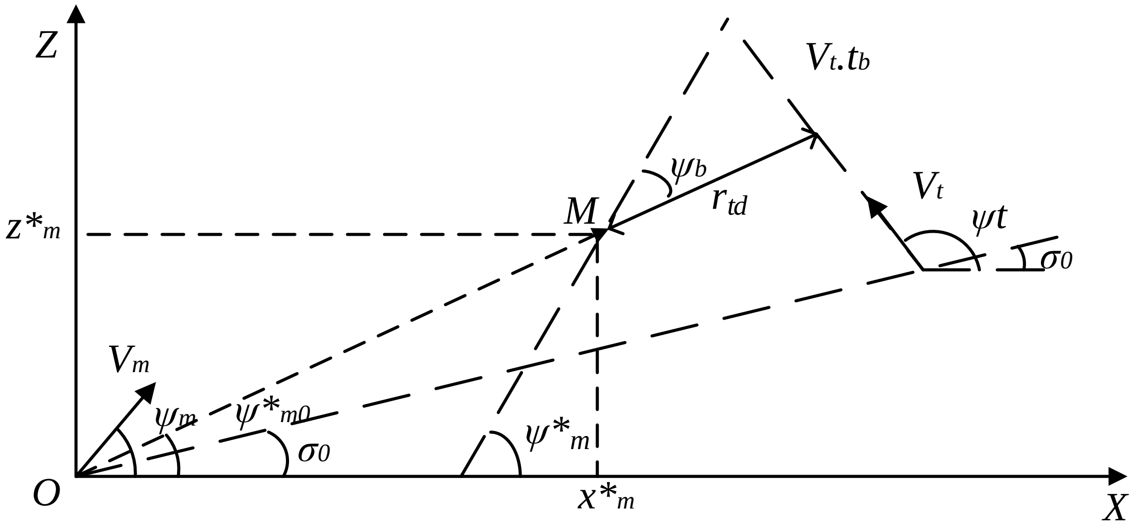

Figure 3. Determine desired heading angle

Assume that the missile moves with a constant velocity V_m from point O to point M along trajectory OM, while the target moves in a straight line with a constant velocity V_t and a trajectory heading angle ψ_t, as illustrated in Figure 3.

Set: $\Delta\psi = \psi^*_{m0} - \sigma_0$; $\Delta\psi_t = \psi_{mt} - \sigma_0$. With the requirement at the moment of switching to autonomous guidance t_{td} : the missile reaches point $M(x^*_m, z^*_m)$;the missile's trajectory heading angle ψ^*_m and the distance from missile to target is r_{td} . From the Figure 3, we have relationship following:

$$\begin{cases}(t_{td}V_m + r_{td})\sin\Delta\psi = t_{td}V_t\sin\Delta\psi_t \\ (t_{td}V_m + r_{td})\cos\Delta\psi = r_0 + t_{td}V_t\cos\Delta\psi_t\end{cases} \tag{19}$$

Squaring both sides of the first and second equations of system (19), then adding the resulting equations side by side, we obtain:

$$(V_m^2 - V_t^2)t_{td}^2 + 2\left(V_m r_{td} - r_0 V_t\cos\Delta\psi_t\right)t_{td} - r_0^2 + r_{td}^2 = 0 \tag{20}$$

The quadratic equation in the variable t_{td} has roots

$$t_{td} = \frac{-\left(V_m r_{td} - r_0 V_t\cos\Delta\psi_t\right) \pm \sqrt{\left(V_m r_{td} - r_0 V_t\cos\Delta\psi_t\right)^2 + (V_m^2 - V_t^2)(r_0^2 - r_{td}^2)}}{V_m^2 - V_t^2}$$

Because time cannot be negative, we take the positive roots:

$$t_{td} = \frac{r_0}{V_m}\frac{-\left(k_r - k_V\cos\Delta\psi_t\right) + \sqrt{\left(k_r - k_V\cos\Delta\psi_t\right)^2 + (1-k_V^2)(1-k_r^2)}}{(1-k_V^2)} \tag{21}$$

With $k_V = V_t / V_m$; $k_r = r_{td} / r_0$.

From the first equation of system (19) there is:

$$\psi_t^* = \arcsin\left(\frac{t_{td}V_t}{t_{td}V_m + r_{td}}\sin\Delta\psi_t\right) + \sigma_0 \tag{22}$$

The coordinat of desired point M obtained:

$$z_m^* = t_{td}V_m \sin\Delta\psi_{m0}^* \tag{23}$$
$$x_m^* = t_{td}V_m \cos\psi_{m0}^*$$

However, when the missile follows the desired trajectory as a Rayleigh function, a compensation term must be considered due to the missile's curved trajectory. Therefore, the desired heading angle of the missile at the end of the autonomous flight phase is given by:

$$\psi_m^* = \psi_{m0}^* + \psi_b \tag{24}$$

When calculating the compensation amount due to the missile's curved trajectory, it is necessary to account for the time deviation when the missile reaches point M compared to a straight trajectory and consider the lead angle required to eliminate the slip error.

*) The time deviation for the missile to reach the interception point is $\mathbf{t_b}$; the initial homing range is $\boldsymbol{r_{td}}$; the compensation lead angle is ψ_b. Based on Figure 3, we obtain

$$\psi_b = \arctan\left(\frac{V_t.t_b.\sin\left(\psi_t - \psi_{m0}^*\right)}{r_{td} + V_t.t_b.\cos\left(\psi_t - \psi_{m0}^*\right)}\right) \tag{25}$$

Where t_b is the compensation time. To determine the compensation time, the following iterative algorithm is implemented:

- Step 1: Set $t_b = \Delta t$ with Δt sufficiently small, obtaining $t_{tdb} = t_{td} + t_b$;

- Step 2, Calculate ψ_b using the following expression (25);

- Step 3, Calculate ψ_m using the following expression (24);

- Step 4, Redefine the Rayleigh function according to $\left(x_{to}^*, z_{to}^*\right),\ \psi_m^*$;

- Step 5, Calculate missle-target distance $r_{M-T}\big|_{t_{tdb}}$ at t_{tdb} ;

- Step 6, Compare, if $\left|r_{td} - r_{M-T}\big|_{t_{tdb}}\right| \le \varepsilon$, finish the programe. If $\left|r_{td} - r_{M-T}\big|_{t_{tdb}}\right| > \varepsilon$, return step 1 with

$t_b = (k+1)\Delta t,\ k = 1 \div n$ (ε- sai số).

) Lead angle to eliminate the slippage:

$$\sin\psi_{tr} = -\frac{V_t}{V_m}\sin(\Delta\psi - \Delta\psi_t) \tag{26}$$

III. RESULT OF SIMULATION

3.1. Survey the guidance law (11) with reference trajectory is Rayleigh funcion

Paramets of survey:

- The missle velocity: $V_m = 900 m/s$;

- Permissible Overload : $n_{zmax} = 30$;

- Permissible time constant: $T_{CP} = 0,5s$;

- Desired trajectory angle at M: $\psi_m^* = 45^0$;

- Desired point M coordinateshe Rayleigh function is considered with the maximum point M_2, passing through M1 in three cases (with distance OM_0= 1000m)

+ Case 1:

$\left(x_1, z_1\right) = \left(-500\sqrt{2},\ 500\sqrt{2}\right)m$, $\left(x_2, z_2\right) = \left(5000,\ 3000\right)m$;

+ Case 2:

$\left(x_1, z_1\right) = \left(-500\sqrt{2},\ 500\sqrt{2}\right)m$, $\left(x_2, z_2\right) = \left(5000,\ 4000\right)m$;

+ Case 3:

$\left(x_1, z_1\right) = \left(-500\sqrt{2},\ 500\sqrt{2}\right)m$, $\left(x_2, z_2\right) = \left(5000,\ 5000\right)m$.

Corresponding to the survey parameters in the three cases above, the initial heading angles of the missile are calculated as follows: $\psi_{m0} = 83,2919^0;\ 92,3536^0;\ 99,5547^0$.

Result of the survey:

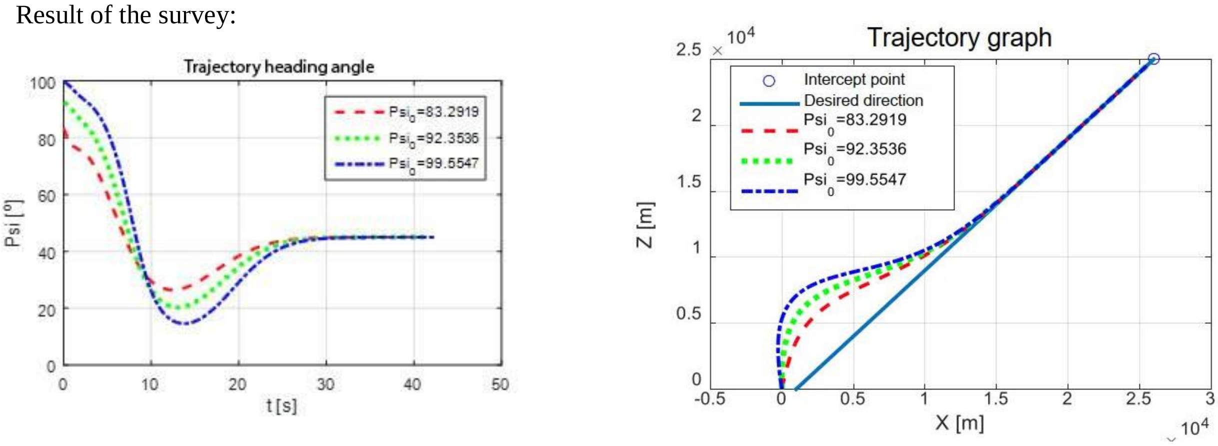

Figure 4. Trajectory and trajectory heading angle based on the Rayleigh reference function

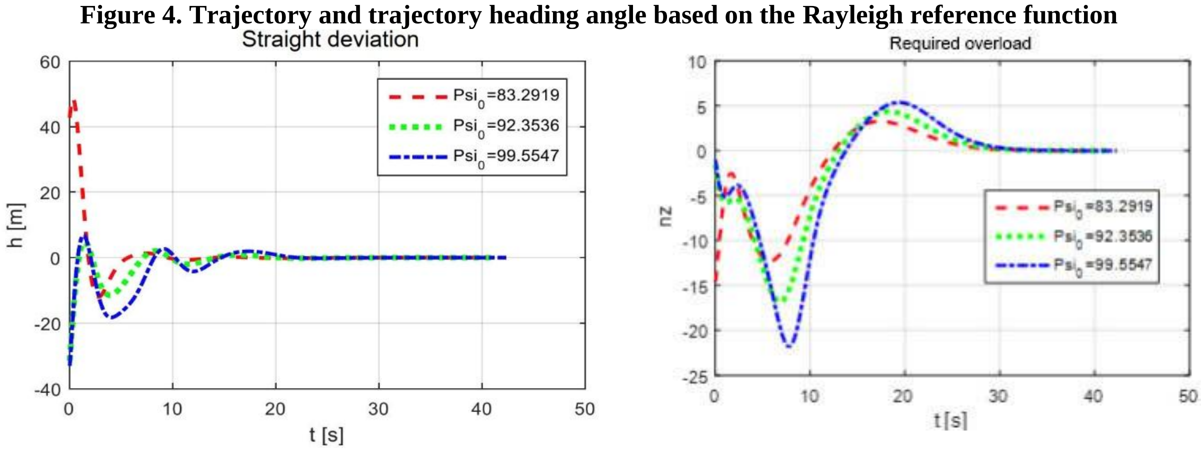

Figure 5. Linear deviation and required overload of the missile

From the simulation results shown in Figures 4 to 5, it can be observed that:

In all three examined cases with a large initial heading angle $q_0 = \psi_{m0} - \psi_m^* = 38,2919^0;\ 47,3536^0;\ 54,5547^0$, the missile's transition time to the steady-state condition is ensured at *t = 30s*. The missile trajectory rapidly converges to the desired direction and passes through the designated target point.

- Depending on the predefined parameters of the Rayleigh reference function, the guidance law (11) generates the tracking overload and eliminates deviations from the reference function. The linear deviation between the missile trajectory and the reference function is rapidly eliminated and reduced to zero. Similarly, the missile trajectory heading angle quickly converges to the desired direction.

3.2. Survey of Desired Target Point Calculation

Survey Parameters:

- Missile velocity: $V_m = 900\text{m/s}$;
- Permissible overload: $n_{zmax} = 30$;
- Permissible time constant: $T_{CP} = 0{,}5\text{s}$;
- Initial line-of-sight inclination angle: $\sigma_0 = 15^0$;
- Initial missile-target distance: $r_0 = 40\ km$;
- Initial homing distance: $r_{td} = 10\ km$;
- Target velocity: $V_t = 300\text{m/s}$;
- Rayleigh function with peak point M2: $(x_2, z_2) = (5, 4)\text{km}$;
- Target heading angle takes the following values $\psi_{mt} = 130^0; 150^0$.

The simulation results are shown in Figure 6

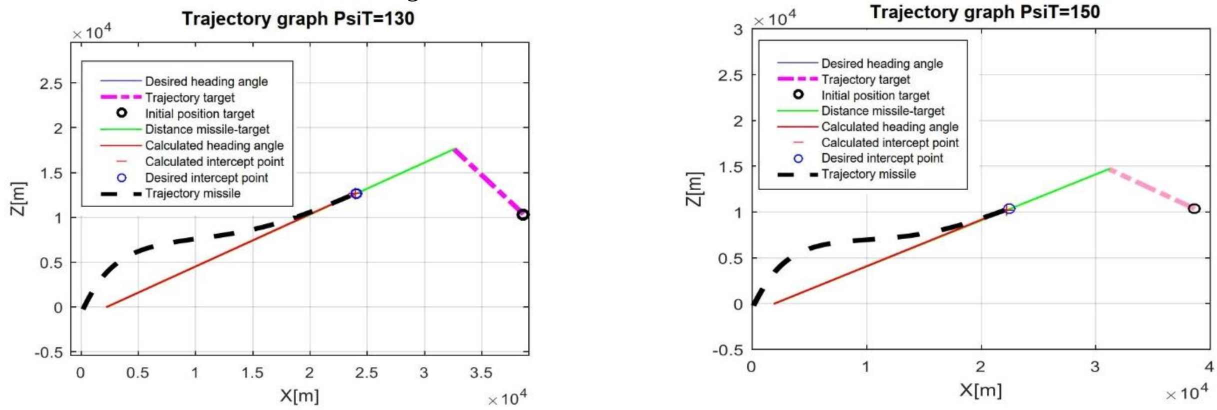

Figure 1. Determination of the desired intercept point

With $\psi_t = 130^0; 150^0$

From the simulation results show on figure 6, with $\psi_t = 130^0$ The desired intercept point is obtained at coordinates $(x_m^*, z_m^*) = (23989, 12620)\text{m}$; The desired heading angle is $\psi_m^* = 30{,}1561^0$. The missile trajectory converges and passes through the desired destination point, ensuring the missile-target distance at the initial homing moment . $r_{td} = 10.000\text{m}$. With $\psi_t = 150^0$, the desired destination point is obtained at coordinates $(x_m^*, z_m^*) = (22431, 10298)m$; with the desired heading angle $\psi_m^* = 26{,}7196^0$. The missile trajectory meets the specified requirements.

IV. CONCLUSION

This paper presents an algorithm for solving the intercept guidance problem, enabling the missile to autonomously navigate to a desired position before transitioning to the homing phase. The requirements at the end of the autonomous flight phase include ensuring the initial homing distance and aligning the missile velocity vector with the missile-target line of sight. The analysis is conducted for a target moving with constant velocity and different initial heading angles, where the missile trajectory follows a Rayleigh function. The results confirm that the proposed algorithm meets the specified requirements.

Conflict of interest

There is no conflict to disclose.

FUTURE DEVELOPMENT

The algorithm can be further developed to address the intercept guidance problem for maneuvering targets, including unidirectional maneuvers or snake-like evasive maneuvers with predefined overloads. This enhancement will bring the problem closer to real-world combat scenarios

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