Research Inventy: International Journal of Engineering And Science
Vol.13, Issue 3 (March. 2023), PP 25-29
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Kulza-Klein perfect fluid cosmological model in f(R,T) theory of gravity

R. N. Patra
P.G. Department of Mathematics, Berhampur University, Odisha, India
raghunathpatra09@gmail.com

Abstract
In this paper we have considered a new class of Kaluza-Klein five dimensional cosmological model filled with perfect fluid in the framework of $f(R,T)$ theory with an appropriate choice of function $f(R,T) = R + 2T$ and time varying deceleration parameter, we have obtained the exact solutions of the field equations. Different characteristics of physical and dynamical properties of the model are discussed by using graphics. Also, we have studied the behaviour of designed cosmological models in different angles with sense of present epoch.
Keywords: *Cosmological model, perfect fluid, f(R,T) theory, Kaluza-Klein, Five dimension, Time varying deceleration parameter*

I. Introduction

In these days, researchers are interested to study alternative theories of gravitation due to existence of the dark energy, dark matter and late time acceleration of the Universe. Out of which f(R,T) theory of gravity proposed by Harko et al., [1] is a special one because it represents the variation of matter stress energy of the metric. Also, for specific choices of the function f(R,T) different cosmological models are presented which gives consistent results.

[9] have investigated on $f(R,T)$ theory using FRW metric and studied on the accelerated Universe. [10] have studied on existence of bulk viscous cosmological model in $f(R,T)$ gravity. Many researchers have studied on $f(R,T)$ gravity using different cosmological models[6], [11]–[15]. [12], [16], [17] have investigated on Kaluza-Klein model using $f(R,T)$ gravity and observed the model in different directions.[18] have studied on higher dimensional cosmological model in the context of Lyra geometry. [12] have discussed on Kaluza-Klein cosmological model in the context of $f(R,T)$ gravity and found in the presence of domain walls.

After Kaluza's work published in 1921, a natural question was raised where is the 5th dimension. This question is very much relevant as we observe only 4-dimensional world. So, if we believe in a 5-dimensional world, the simplest approach is to think that the extra dimension (5th dimensional) is too small to observe. It indicates that extra dimension should be compact.

To find the exact solution of Einstein field equation, we have assumed a special form of volume. In sect. 2: the field equation of $f(R,T)$ gravity, in sect. 3: metric and field equations and their solution and sect. 4 deals with conclusion of the study.

II. Field equation of $\mathbf{f(R,T)}$ gravity

The $f(R,T)$ theory of gravity is a modification of General Relativity. The field equations are derived from the Hilbert-Einstein type variation principle. The action for the modified $f(R,T)$ gravity is

$$S= \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^5x + \int L_m\sqrt{-g}d^5x \tag{1}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar (R) and be the trace of stress–energy tensor T_{ij} of the matter(T). L_m is the matter Lagrangian density. The energy momentum tensor T_{ij} is defined is

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}\right)L_m}{\partial g^{ij}} \tag{2}$$

and its trace by $T = g^{ij}\, T_{ij}$. Here, we assume that the dependence of matter Lagrangian is merely on the metric tensor g_{ij} rather than its derivatives. In this case, we obtain

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}} \tag{3}$$

The $f(R,T)$ gravity field equations are obtained by changing the action of S with respect to metric tensorg_{ij}.

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^{\mathrm{i}}\nabla_{\mathrm{i}} - \nabla_{\mathrm{i}}\nabla_{\mathrm{j}}\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \tag{4}$$

Where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\propto\beta}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{\propto\beta}} \quad (5)$$

$f_R(R,T) = \frac{\partial f(R,T)}{\partial R}, f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$, where the ∇_i stands for covariant differentiation. Now contraction of equation (4) gives

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + 3(\nabla_i\nabla^i)f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)(T+\theta), \quad (6)$$

where $\theta = \theta_i^i$. Equation (4) gives a relation between Ricci scalar R and trace T of energy momentum tensor. Using matter Lagrangian L_m, the energy momentum tensor for a one fluid source.

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij,} \quad (7)$$

where $u^i = (0,0,0,0,1)$ is the velocity vector in co-moving coordinates which satisfies the condition $g^{ij}u^i u_i = -1$ and $u^i\nabla_j u_i = 0$. ρ and p are energy density and isotropic pressure of the fluid respectively and the matter Lagrangian can be taken as $L_m = -p$.

The stress energy of perfect fluid is

$$\theta_{ij} = -2T_{ij} - pg_{ij} \quad (8)$$

The $f(R,T)$ theory of gravity depending on the nature of the matter source. We take

$$f(R,T) = R + 2f(T). \quad (9)$$

In this paper, we have considered $f(R,T) = R + 2T$.

The gravitational field equation of $f(R,T)$ gravity from equation (4) gives

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2)T_{ij} + (2p + T)g_{ij}. \quad (10)$$

Einstein field equations with cosmological constant is expressed as

$$G_{ij} - \Lambda g_{ij} = -8\pi T_{ij}. \quad (11)$$

From equation (10) and equation (11) we obtained

$$\Lambda = 2p + T \quad (12)$$

III. Metric and Field Equations

Consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\psi^2 \quad (13)$$

Here the fourth coordinate ψ is taken to be space-like and A, B are function of cosmic time t only.

The field equation (10)-(12) for the metric(13) with the help of energy-momentum tensor (7) gives the following system of equations

$$\left(\frac{A''}{A}\right) + 2\left(\frac{A'}{A}\right)^2 + \frac{A'B'}{AB} - \frac{3}{2}\left(A^3A'' + \frac{A''}{A} + \frac{A'B'(A^4+B^4)}{AB}\right) - 3A^2A'^2 - \frac{1}{2}\left(B^3B'' + \frac{B''}{B}\right) = (8\pi - 2)p + (8p + \rho) \quad (14)$$

$$3\left(\frac{A'B'}{AB}\right) - \frac{3}{2}\left(A^3A'' + \frac{A''}{A} + \frac{A'B'(A^4+B^4)}{AB}\right) - 3A^2A'^2 - \frac{1}{2}\left(B^3B'' - \frac{B''}{B}\right) = (8\pi - 2)p + (8p + \rho) \quad (15)$$

and $\frac{3}{2}\left(\frac{A''}{A} - A^3A'' - \frac{A'B'(A^4+B^4)}{AB}\right) - 3A^2A'^2 - \frac{1}{2}\left(B^3B'' - \frac{B''}{B}\right) = (8\pi - 2)(2p + \rho) + 3(4p + \rho)$. (16)

Subtracting equation (14) from (16)

$$\frac{B''}{B} - \frac{A'B'}{AB} + 2\left(\frac{A''}{A}\right) - 2\left(\frac{A'}{A}\right)^2 = (8\pi - 2)(p + \rho) + (4p + 2\rho) \quad (17)$$

Subtracting equations (15) from (16)

$$3\frac{A''}{A} - 3\frac{A'B'}{AB} = (8\pi - 2)(p + \rho) + (4p + 2\rho) \quad (18)$$

Subtracting (17) from (18)

$$\frac{A''}{A} + 2\left(\frac{A'}{A}\right)^2 - 2\frac{A'B'}{AB} - \frac{B''}{B} = 0. \quad (19)$$

Where an overhead prime denotes ordinary derivative with respect to cosmic time 't' only.

Rewriting the equation (19) is of the form

$$\frac{d}{dt}\left(\frac{A'}{A}-\frac{B'}{B}\right)+\left(\frac{A'}{A}-\frac{B'}{B}\right)\frac{V'}{V}=0. \quad (20)$$

Here we considered the spatial volume

$$V=A^3B. \quad (21)$$

Integrating both side of equation (20) w.r.to. t we get

$$\frac{A}{B}=c_2e^{c_1\int\frac{dt}{V}}. \quad (22)$$

Using equation (21) & (22) the metric potentials can be represented as

$$A=c_2^{1/4}V^{1/4}e^{c_1/4\int\frac{dt}{V}} \quad (23)$$

and

$$B=c_2^{-3/4}V^{1/4}e^{-3c_1/4\int\frac{dt}{V}}. \quad (24)$$

Where $c_2^{1/4}$ and $c_2^{-3/4}$ are integration constant, let us consider $c_2^{1/4}=\beta$ and $c_2^{-3/4}=\delta$.

Using equation (23) and (24) the resulting solution of field equations (17) and (18) can be expressed in terms of V as follows

$$\frac{3}{4}V^{-2}(c_1^2-1)=(8\pi-2)(p+\rho)+(4p+2\rho)$$

IV. Physical properties:

The directional **Hubble parameters** along different directions are defined as

$$H_x=H_y=H_z=\frac{A'}{A}\ \&H_\psi=\frac{B'}{B}.$$

To treat the metric potentials in a physically realistic way, here we have considered the volume of the Universe as a power law form i.e.

$$V=A^3B=t^be^{kt} \quad (25)$$

Here we took $b=0$, which covers all possible expansion histories throughout the evolution of the universe. The positive nature of the exponent is in accordance with observational findings predicting an expanding universe. The power law expansion of the universe predicts a deceleration parameter $q=-\frac{a\ddot{a}}{(\dot{a})^2}$, where a stands for the average scale factor of the universe and is related to the directional scale factors A & B through $a=(A^3B)^{\frac{1}{4}}$.

The **directional scale factors** can be obtained by using (25) in (23) & (24) as

$$A=c_2^{\frac{1}{4}}(e^{kt})^{\frac{1}{4}}e^{\frac{c_1}{4}\left(\frac{e^{-kt}}{-k}\right)}, \quad (26)$$

$$B=c_2^{-\frac{3}{4}}(e^{kt})^{1/4}e^{\frac{-3c_1}{4}\left(\frac{e^{-kt}}{-k}\right)}, \quad (27)$$

$$H_x=\frac{(1+c_1)}{4}(e^{-kt}) \quad (28)$$

$$H_\psi=\frac{(1-3c_1)}{4}(e^{-kt}) \quad (29)$$

$$H=\frac{3H_x+H_\psi}{4}=\frac{1}{4}(e^{-kt}). \quad (30)$$

The directional Hubble parameters in respective directions are

$$H_x=H_y=H_z=H+\frac{c_1}{4}(e^{-kt}) \quad (31)$$

and $H_\psi=H-\frac{3c_1}{4}(e^{-kt})$. (32)

The anisotropy parameter Δ is

$$\Delta=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{(H_i-H)}{H}\right)^2=3c_1^2, \quad (33)$$

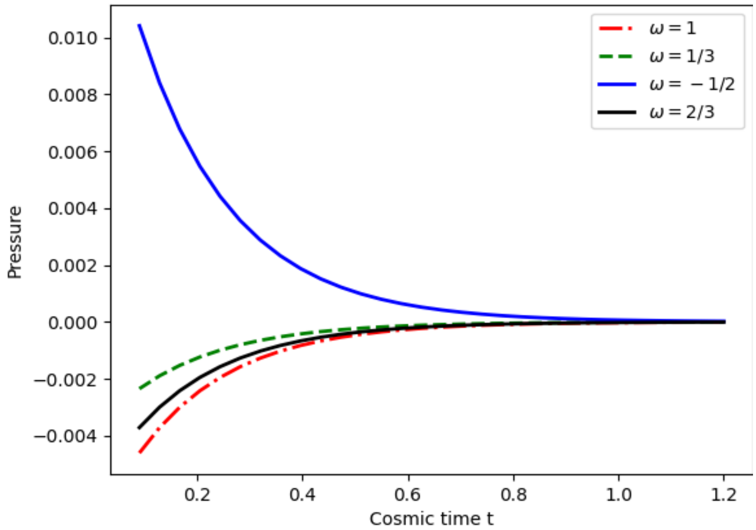
where $H_i\ (i=1,2,3,4)$ represent the directional Hubble parameters in the directions of x, y, z & ψ respectively.

The **expansion scalar** $\theta = 4H = e^{-kt}$ (34)
and the shear **scalar** $\sigma^2 = 6c_1^2 e^{-2kt}$. (35)
The other unknowns are obtained as follows

$$P = \frac{3[(c_1^2-1)\omega e^{-2kt}]}{32\pi(\omega+1)+8\omega} \quad (36)$$

$$\rho = \frac{3[(c_1^2-1)e^{-2kt}]}{32\pi(\omega+1)+8\omega} \quad (37)$$

Case-I for $\omega = 0$ We get $p = 0$.(Dust) (38)
Case-II for $\omega = 1$ We get $p = \rho$ (stiff matter) (39)
Case-III for $\omega = 1/3$ We get $p = \frac{1}{3}\rho$ (radiating universe) (40)
Case-IV for $\omega < -1/3$ We get $p = -\frac{1}{2}\rho, by\ taking\ \omega = -\frac{1}{2} we\ verified$ (Accelerated expansion of the universe) (41)
Case-V for $\omega \in (\frac{1}{3}, 1)$ Hard Universe (42)



The observational data
The deceleration parameter q ,the jerk parameter j,the snap parameter $s^*$and the lerk parameter l of our model is obtained as

The deceleration parameter $q = -\frac{a\ddot{a}}{(\dot{a})^2} = -1$. (43)

The jerk parameter $j = \frac{a^2\dddot{a}}{(\dot{a})^3} = 1$ (44)

Snap Parameter $s^* = -\frac{a^3\ddddot{a}}{(\dot{a})^4} = -1$ (45)

The expansion of a(t) around a(0) using taylor's theorem as :

$$a(t) = a(0) + \dot{a}(0)t + \frac{\ddot{a}(0)}{2!}t^2 + \frac{\dddot{a}(0)}{3!}t^3 + \cdots \quad (46)$$

Let $a(t) = a\ and\ a(0) = a_0$,we get

$$a = a_0 + \dot{a}_0 t + \frac{\ddot{a}_0}{2!}t^2 + \frac{\dddot{a}_0}{3!}t^3 + \cdots$$

Or

$$\frac{a}{a_0} = 1 + \frac{\dot{a}_0}{a_0}t + \frac{\ddot{a}_0}{2!a_0}t^2 + \frac{\dddot{a}_0}{3!a_0}t^3 + \cdots \quad (47)$$

The current values of the parameters and appear in the Taylors expansion(48) around $a_0$as

$$\frac{a}{a_0} = 1 + H_0 t - \frac{q_0 H_0^2}{2!}t^2 + \frac{q_0 H_0^3}{3!}t^3 + \cdots \quad (49)$$

V. Conclusion:

In this paper, the role of perfect fluid with cosmological constant is studied for deriving perfect fluid solutions in view of five-dimensional Kaluza-Klein space-time in gravity corresponding to five different cases of EoS parameter. In first case i.e., when it is shown that the obtain model of the universe is vacuum which is otherwise known as dust distribution, in case-2 i.e., when the obtained model of the universe is isotropic. Further, in case-3, we obtained a radiating universe and in case-4 we obtained a universe where pressure remains negative and energy density remains positive which yields an accelerating and expanding universe which ultimately gives a flat space-time. Here the pressure initially diverging in nature and later on it vanishes. The obtained universe is anisotropic in nature throughout the entire evolution. It is also observed that at the current stage the universe is expanding in nature and will collapse at infinite future.

References:

[1]. Planck Collaboration et al., "Planck 2018 results. I. Overview and the cosmological legacy of Planck," 2018.

[2]. E. Macaulay et al., "First cosmological results using Type Ia supernovae from the Dark Energy Survey: measurement of the Hubble constant," Mon. Not. R. Astron. Soc., vol. 486, no. 2, pp. 2184–2196, Jun. 2019.

[3]. T. R. Jaffe, A. J. Banday, H. K. Eriksen, K. M. Górski, and F. K. Hansen, " Bianchi type VII h models and the WMAP 3-year data ," Astron. Astrophys., vol. 460, no. 2, pp. 393–396, 2006.

[4]. S. Perlmutter et al., "Measurements of O and L from 42 High-Redshift," Astrophys. J., vol. 517, pp. 565–586, 1999.

[5]. R. J. Adler, H. Mueller, and M. L. Perl, "A terrestrial search for dark contents of the vacuum, such as dark energy, using atom interferometry," pp. 1–25, 2011.

[6]. S. Bahamonde, C. G. Böhmer, and M. Kršśák, "New classes of modified teleparallel gravity models," Phys. Lett. Sect. B Nucl. Elem. Part. High-Energy Phys., vol. 775, 2017.

[7]. G. Lyra, "Uber eine Modifikation der Riemannschen Geometrie," Math. Zeitschrift, vol. 54, no. 1, pp. 52–64, 1951.

[8]. N. Bilić, G. B. Tupper, and R. D. Viollier, "Dark Matter, Dark Energy and the Chaplygin Gas," in Dark Matter in Astro- and Particle Physics, Berlin, Heidelberg: Springer Berlin Heidelberg, 2002, pp. 306–311.

[9]. R. Myrzakulov, "FRW cosmology in F(R,T) gravity," Eur. Phys. J. C, vol. 72, no. 11, p. 2203, Nov. 2012.

[10]. M. Kiran and D. R. K. Reddy, "Non-existence of Bianchi type-III bulk viscous string cosmological model in f(R,T) gravity," Astrophys. Space Sci., vol. 346, no. 2, pp. 521–524, 2013.

[11]. S. D. Katore, S. P. Hatkar, and R. J. Baxi, "Domain wall cosmological models with deceleration parameter in modified theory of gravitation," Chinese J. Phys., vol. 54, no. 4, pp. 563–573, 2016.

[12]. A. K. Biswal, K. L. Mahanta, and P. K. Sahoo, "Kaluza-Klein cosmological model in f (R , T) $f(R,T)$ gravity with domain walls," Astrophys. Space Sci., vol. 359, no. 2, p. 42, Oct. 2015.

[13]. M. F. Shamir, "Plane Symmetric Solutions in f(R,T) Gravity," Commun. Theor. Phys., vol. 65, no. 3, 2016.

[14]. P. K. Agrawal and D. D. Pawar, "Magnetized domain wall in f(R, T) theory of gravity," New Astron., vol. 54, 2017.

[15]. E. Güdekli, N. Myrzakulov, K. Yerzhanov, and R. Myrzakulov, "Trace-anomaly driven inflation in f(T) gravity with a cosmological constant," Astrophys. Space Sci., vol. 357, no. 1, 2015.

[16]. D. D. Pawar, G. G. Bhuttampalle, and P. K. Agrawal, "Kaluza–Klein string cosmological model in f(R, T) theory of gravity," New Astron., vol. 65, 2018.

[17]. P. K. Sahoo, B. Mishra, and S. K. Tripathy, "Kaluza–Klein cosmological model in f(R, T) gravity with Λ(T)," Indian J. Phys., vol. 90, no. 4, pp. 485–493, Apr. 2016.

[18]. F. RAHAMAN and JAYANTA KUMAR BERA, "HIGHER DIMENSIONAL COSMOLOGICAL MODEL IN LYRA GEOMETRY: REVISITED," Int. J. Mod. Phys. D, vol. 12, no. 05, pp. 853–860, May 2003.