Research Inventy: International Journal of Engineering And Science
Vol.12, Issue 8 (August 2022), PP 01-04
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Chains/Necklaces for a Wide Range of Algebraic Operations on The Two Consecutive Numbers Restricted to a Specified Set of Numbers

Tejmal Rathore, IEEE LSM
Independent researcher
G 803 Country Park, Dattapada Road, Borivali (East), Mumbai 400066, India
Corresponding Author:Tejmal Ratore

Abstract:*Rathore has proposed a systematic method of obtaining a chain/necklace when the sum of two consecutive numbers is a perfect square or cube. This method is extended to a wide range of algebraic operations on the two consecutive numbers;but resticted to a set of numbers.*
Keywords:*Chain, Necklace, Magic circle, Rathore Method, Perfect square*

Date of Submission: 12-08-2022 Date of Acceptance: 28-08-2022

I. INTRODUCTION

Rathore has presented method of generating necklaces of numbers 1-*n* with the sum of any two consecutive numbers either a perfect square [1] or a perfect cube [2]. Also, necklaces with the absolute value of the difference of two consecutive numbers as a perfect square are derived [3]. In case of perfect square case, the sum or difference of the two consecutive numbers is restricted to the numbers to 1, 4, 9, 16, 25, 36, 49, …[1]. In case of perfect cube, the sum is restricted to the numbers 1,8,27,64,125,216,343,512, …[2]. In this paper, we extend the method to a wide range of *algebraic operations* on the two consecutive numbers; restricted to a given set of numbers.

II. METHOD

Let a, b and c ($a \neq b \neq c$) be three consecutive numbers on the chain/necklae. Then algebraic operation on a and b, and b and c $=$ should be numbers from a specified set.

2.1 Choices for *b*

We find it convenient to demonstrate the choices for *b for the restriction that the sum of two neighbouring numbers is a perfect square* for*n* = 36 through Table 1.
The choices can be divided into four categories.

1. If there is *no choice* for a number, the number will appear as an isolated number.
2. If there is *a single choice* for *b*, then *b* will appear at the end of a chain.

Table 1. Choices for *b* for *n*-36

1-3,8,15,24,35	10-6,15,26	19-6,17,30	28-8,21,36
2-7,14,23,34	11-5,14,25	20-5,16,29	29-7,20,35
3-1,6,13,22,33	12-4,13,24	21-4,15,28	30-6,19,34
4-5,12,21,32	13-3,12,23	22-3,14,27	31-5,18,33
5-4,11,20,31	14-2,11,22	23-2,13,26	32-4,17
6-3,10,19,30	15-1,10,21	24-1,12,25	33-3,16,31
7-2,9,18,29	16-9,20,33	25-11,24	34-2,15,30
8-1,17,28	17-8,19,32	26-10,23	35-1,14,29
9-7,16,27	18-7,31	27-9,22	36-13,28

3. If there are *onlytwo* options *a* and *c* for *b*, then (*a*,*b*,*c*) will be termed as a *Trio* (T). See number 18 in Table 1. It is possible that any one of the end numbers (*a* or *c*) of a T may not have any other possible option. This number will also appear as an end number of a chain.
4. If there are *more than* options,then one has to make appropriate choice for *b.*

2.2 Some Useful Properties

1. If a T (*a*,*b*,*c*) is such that *a* + *c* is also a perfect square, then (*a*,*c*,*b*) is also a T. It means *b* and *c* can be interchanged.
2. If two strings (Ss) are {*a,b,c,d, … x,y,z*} and {*c,d,…,x,y,z*}, then latter can be ignored.
3. Merging property: Two Ss {*a,b,…,p,q,v*} and {*p,q,v,..,w,x,y,z*} can be merged in to a single S {*a,b,…,p,q,v,..w,x,y,z*}.
4. Mixing property: If two Ss are {*a,b,c,d,…,h*} and {*i,j,…w,x,y,z*} are such that *h* + *i*satisfies the specified critarion,they can be replaced by a single S {*a,b,c,d,…,h,i,,j,….,w,x,y,z*}

2.3 The steps of the method

(1) Prepare a Table for the given *n* for the choices of *b* for the specified algebraic operation and the given set of numbers such as Table 1 for square values.
(2) List all Ts for the given *n*. These Ts have to appear in the chain(s) and the necklace(s).
(3) Eliminate the common numbers present in various Ts using properties 2 and 3. This step will give number of strings (Ss) shown inside {} brackets.
(4) Attach *all* the left-out numbers (LONs) around various Ss such that the given criterion is maintained. If it is not possible, then the full chain or full necklace does not exist.
(5) Interconnect all the Ss using the property 4. There may or may not be a full necklace.

In [1], the sum of two consecutive numbers was restricted to the set of numbers (perfect square) 4,9,16,25,36,49,64, .. , and in [2], the set of numbers (perfect cube) is 8,27,64,125,216,343,512,729,1000,… Obviously, it should be possible to extend the method to any other set of numbers. Few illustrative examples are given in the next section.

2.4 Illustrative examples

2.4.1 Example 1:*n* = 60, and the *sum* of two consecutive numbers is restricted to 10,20,30,40,50,60.
Table 2 shows the options for the numbers 1-60. Note that 60 has no option, thus, it will be an isolated number. Numbers 50-59 have one option. They will appear at the end of chains. Hence, there will definitely be chain(s) and no necklace.
Trios are (10,40,20), (9,41,19),(8,42,18),(7,43,17),(6,44,16), (5,45,15),(4,46,14),(3,47,13),(2,48,12),(1,49,11)
Eliminating the common numbers from the trios, we get(10,40,20),(19,41,9,1,49,11),(18,42,8,2,48,12),(17,43,7,3,47,13),(16,44,6,4,46,14),(5,45,15).
LONs are21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,50,51,52,53,54,55,56,57,58,59,60. Attaching LONs, we get

Table 2. Choices for *b* for *n* 60.

1-9,19,29,39,49,59	13-7,17,27,37,47	25-5,15,25,35	37-3,13,23	49-1,11
2-8,18,28,38,48,58	14-6,16,26,36,46	26-4,14,24,34	38-2,12,22	50-10
3-7,17,27,37,47,57	15-5,15,25,35,45	27-3,13,23,33	39-1,11,21	51-9
4-6,16,26,36,46,56	16-4,14,24,34,44	28-2,12,22,32	40-10,20	52-8
5-5,15,25,35,45,55	17-3,13,23,33,43	29-1,11,21,31	41-9,19	53-7
6-4,14,24,34,44,54	18-2,12,22,32,42	30-10,20,30	42-8,18	54-6
7-3,13,23,33,43,53	19-1,11,21,31,41	31-9,19,29	43-7,17	55-5
8-2,12,22,32,42,52	20-10,20,30,40	32-8,18,28	44-6,16	56-4
9-1,11,21,31,41,51	21-9,19,29,39	33-7,17,27	45-5,15	57-3
10-20,30,40,50	22-8,18,28,38	34-6,16,26	46-4,14	58-2
11-9,19,29,39,49	23-7,17,27,37	35-5,15,25	47-3,13	59-1
12-8,18,28,38,48	24-6,16,26,36	36-4,14,24	48-2,12	60-

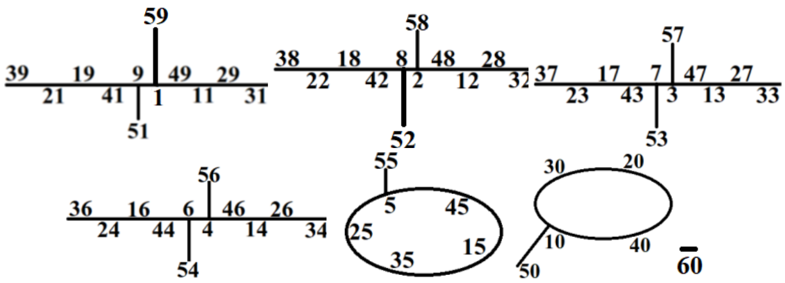

Figure 1. CND forExample 1

{39,21(19,41,9,1,49,11),29,31},{38,22,(18,42,8,2,48,12),28,32},{37,23,(17,43,7,3,47,13),27,33}, {36,24,(16,44,6,4,46,14),26,34},{25,(5,45,15),35}, {50,(10,40,20),30},{51,9}, {52,8}, {53,7}, {54,6}, {55,5}, {56,4}, {57,3}, {58,2}, {59,1}, 60.These Ss cannot be interconnected. Therefore, we get 1 isolated number (60),14 chains and 2 necklaces.The chain-necklace diagram (CND) is shown in Fig. 1.

2.5.2 Example 2: Let *n* = 23, and the *sum* should be from *the numbers with exponent greater than or equal to* 3, i.e., 8,16,27,32, ...
Table 3 gives the choices for this case.

Table 3. Choices for numbers 1-23

1-7,15	7-1,9,20	13-3,14,19	19-8,13
2-6,14	8-19	14-2,13,18	20-7,12
3-5,13	9-7,18,23	15-1,12,17	21-6,11
4-12,23	10-6,17,22	16-11	22-5,10
5-3,11,22	11-5,16,21	17-10,15	23-4,9
6-2,10,21	12-4,15,20	18-9,14	

Trios are7,1,15),(6,2,14),(5,3,13),(12,4,23),(10,17,15),(9,18,14),(8,19,13),(7,20,12),(6,21,11),(5,22,10),(4,23,9). Eliminating the common numbers in the trios, we get(11,21,6,2,14,18,9,23,4,12,20,7,1,15,17,10,22,5,3,13,19,8). Attaching the LON 16 to the above string, we get a full chain16,11,21,6,2,14,18,9,23,4,12,20,7,1,15,17,10,22,5,3,13,19,8. This is shown in Fig. 2.
It has been verified, from this chain of 23 by removing lower numbers one by one and rearranging the remining numbers, that all the numbers below 23 have more than 1 chains. Hence, 23 is the minimal for a full chain.

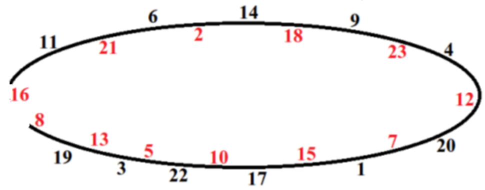

Figure 2. CND for Example 2

2.5.3 Example 3: Let *n* = 9 and the *sum* of two consecutive numbers is restricted to the *triangular series* 1,3,6,10,15,…

Table 4 gives the possible choices for numbers 1-9.

Table 4. Choices of numbers 1-9

1-2,5,9	4-6	7-3,8
2-1,4,8	5-1	8-2,7
3-7	6-4,9	9-1,6

Trios are (4,6,9),(3,7,8),(2,8,7),(1,9,6).Eliminating the common numbers from the trios, we get (4,6,9,1,5),(2,8,7,3).Interconnecting the strings, we get a full chain 3,7,8,2,4,6,9,1,5 and shown in Fig. 3.

3 7 8 2 4 6 9 1 5

Figure 3. CND for Example 3

All the numbers less than 9 have more than one chains. Hence 9 is the minimal for the full chain.

2.5.4 Example 4: Arrange the numbers 1-20 such that the absolute value of *the difference of two adjacent numbersis neither a square nor a prime number*.

Square numbers in the range 1-20 are 1,4,9,16 and the prime numbers are 1,2,3,5,7,11,13,17,19. Therefore, the numbers allowed are 6,8,10,12,14,15,18. The list of options for natural numbers 1-20 is given in Table 5. Trio is 1,14,4. Attaching the LONs 1,2,3,4,5,7,9,11,13,16, we get{9,6,8,7,5,13,2,12,3,11,(1,14,4),10}, {15,3}, {16,2}, {17,1}, {18}. {19}, {20}. CND isshown in Fig. 4.

Table 5. Choices for numbers 1-20

1-5,7,9,11,13,14,17	6-2,4,8,9,12	11-1,3,4,7	16-2
2-4,6,8,10,12,13,16	7-1,3,5,8,11	12-2,3,6	17-1
3-5,7,9,11,12,15	8-2,4,6,7,10	13-1,2,5	18-
4-2,6,8,10,11,14	9-1,3,5,6,9	14-1,4	19-
5-1,3,7,9,10,13	10-2,4,5,8	15-3	20-

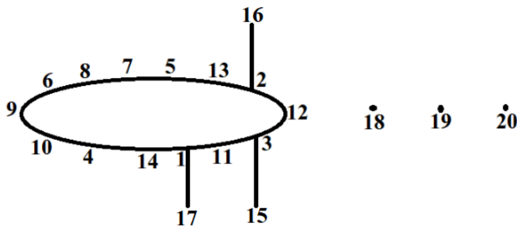

Figure 4. CND for Example 4

2.5.5 Example 5: Let *n* = 20 and the prtoduct be a perfect square (1,4,9,16,25,36,49,64,…).

The choices for the numbrs1-20 are shown in Table 6.

Table 6. Choices for numbers 1-20

1-4,9,16	6-	11-	16-1,4,9
2-8,18	7-	12-3,18	17-
3-12	8-2	13-	18-2,8,12
4-1,9,16,	9-4	14-	19-
5-20	10-	15-	20-5

From Table 6, we note that 6,7,10,11,13,14,15,17,19 are isolated points. Trio ares (8,2,18), (3,12,18). Eliminating the common numbers, we get the S (8,2,18,12,3).LONsare 1,4,5,9,16. Inserting the LONs around the Ss, we get {8,2,18,12,3},{16,4,9,1}, {5,20}. CND is shown in Fig. 5.

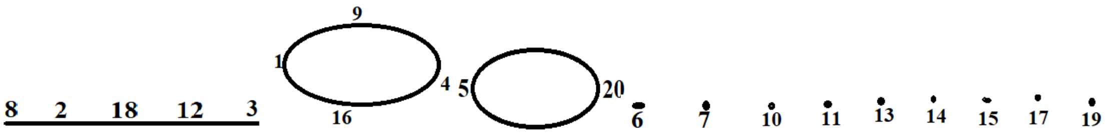

Figure 5. 1 CNDfor Example 5

III. CONCLUSION

Rathore has proposed a systematic method of obtaining a chain/necklace when the sum of two consecutive numbers is a perfect square or cube. This method has been extended to a wide range of algebraic operations such as sum, difference, productetc. on the two consecutive numbers; but resticted to a given set of numbers. The method has been illustrated with several examples.

Conflict of interest

There is no conflict to disclose.

REFERENCES

[1]. Rathore, Tejmal, 2022, Arranging integer numbers on a loop such that the sum of any two adjacent numbers is a perfect square, IEEE Region 10 Symposium, IIT Bombay, July 1-3. Presented.

[2]. Rathore, Tejmal, 2022, Chain-necklace diagrams with the sum of two consecutive numbers a perfect cube, AKGEC International Journal of Technology, Vol. 13, No. 2, in press.

Tejmal Rathore, IEEE LSM. "Chains/Necklaces for a Wide Range of Algebraic Operations on The Two Consecutive Numbers Restricted to a Specified Set of Numbers." *International Journal of Engineering and Science*, vol. 12, no. 8, 2022, pp. 01-04.